Probability density function method for variable-density turbulence and mixing

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A probability density function (PDF) method [1, 2] is developed to compute variable-density (VD) pressure-gradient-driven turbulence. The method computes the joint PDF of density and velocity in a mixture of two different-density molecularly mixing fluids. The model captures the full time-evolution of the joint PDF in a highly non-equilibrium flow in a time-accurate manner, starting from a quiescent state, transitioning to turbulence and finally dissipated by molecular diffusion.

In such binary mixing problems if the densities of the fluids are vastly different, the pressure field applied to very-different-density fluids results in several non-negligible VD effects over the Boussinesq case: cubic non-linearities in the Navier-Stokes equation, dynamic mean pressure gradient and an asymmetric mixing process evidenced by a skewed density PDF [3]. One of the consequences of these new VD effects is the dynamic importance of the specific volume and its correlations, required to close the moment equations of hydrodynamics and mixing. In the joint PDF method, both hydrodynamic turbulence and mixing are treated at arbitrary density ratios, with the specific volume and all its correlations in closed form.

The generalized Langevin model [4], originally developed for the Lagrangian fluid particle velocity in shear-driven turbulence, is extended to pressure-gradient-driven flows representing the mass flux without closure assumptions. The persistent small-scale anisotropy, a fundamentally *"non-Kolmogorovian"* feature of externally accelerated, e.g. buoyantly driven, flows is captured by a tensorial diffusion term based on the buoyancy force. A model for active material mixing is also developed based on the density field, represented by a stochastic differential equation that yields a beta distribution. This enables the joint model to capture the essential features of VD mixing, such as the mixing asymmetry due to large differential fluid accelerations.

To date statistical models cannot predict any of the above features. Compared to most turbulence models developed for equilibrium flows, the PDF method surprisingly captures the time-accurate representation of the highly non-linear process of transition to turbulence. In addition, the joint model reproduces both the large and small scale Reynolds stress anisotropy without resorting to gradient diffusion hypotheses, and represents the mixing state by the density PDF itself, eliminating the need for dubious mixing measures. The model is validated against data obtained from the recent direct numerical simulations of the homogeneous Rayleigh-Taylor instability by Livescu & Ristorcelli [5, 6].

References

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