TENSION-DRIVEN RICHTMYER-MESHKOV INSTABILITY

Hugh Lund & Stuart B. Dalziel

s.dalziel@damtp.cam.ac.uk
Department of Applied Mathematics and Theoretical Physics
University of Cambridge, Wilberforce Road, Cambridge CB3 0WA, UK

The relationship between gravity waves on a statically stable interface and Rayleigh-Taylor instability on a statically unstable interface is well known. In the former case, the acceleration field creates a body force that continually acts to return the interface to horizontal, with inertia leading to an overshoot, inverting the phase of the waves, and thus giving rise to an under-damped oscillation. In the latter case Rayleigh-Taylor instability has the sign of the restoring force reversed, leading to a monotonically growing solution rather than an oscillating one.

Although we tend to think of Richtmyer-Meshkov instability as the result of an impulsive acceleration of a density interface, the flow is essentially equivalent to that which arises from gravity waves on an interface when the restoring force (*i.e.* gravity) is instantaneously turned off at $t = 0$. For $t < 0$ there is an equipartition between kinetic and gravitational potential energy (averaged over a wave period). At $t = 0$ gravitational potential energy ceases to exist, but the instantaneous kinetic energy remains. If $t = 0$ corresponds to a flat interface for a single mode linear standing wave then the kinetic energy will be maximum and the velocity field will have the same form as an impulsive acceleration of the interface between incompressible fluids with infinitesimal perturbations (of the same wavenumber) to the interface. (Of course, if $t = 0$ corresponds to the maximum amplitude for the standing wave, then there is no kinetic energy and the disturbance will not evolve in the absence of a restoring force.)

In this paper we consider a new form of instability. As with Richtmyer-Meshkov instability, the flow develops from a field of waves where the restoring force is suddenly removed. In particular, we look at the rupturing of a water-filled balloon. Before its rupture, the elastic tension in the latex membrane of the balloon supports capillary-like waves on the surface, with an equipartion between kinetic energy of the fluid and elastic energy of the membrane. One half of this equipartition is instantaneously removed when the membrane ruptures. The kinetic energy of the capillary-like waves is no longer balanced by changes in surface energy, leading to (in the absence of gravity and surface tension) unconstrained growth of the disturbances. In the high Atwood number case (water-filled balloon in air), the instability takes the asymmetric form of bubbles and spikes expected for Richtmyer-Meshkov instability (see figure 1).

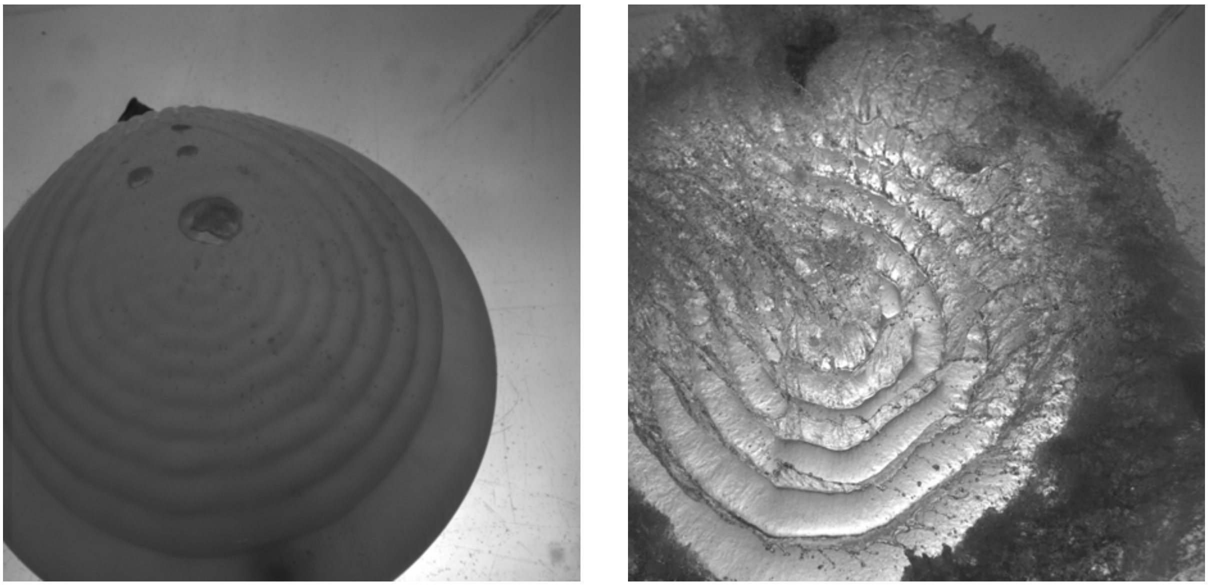

Figure 1: The rupturing of a water-filled balloon. (a) The capillary-like waves following collision with a rigid surface. (b) The Richtmyer-Meshkov-like instability following the removal of the restoring force.