Numerical investigation of initial stage of the vortex cascades

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PROBLEM. The phenomenon of turbulence is complicated and multilateral. There are a great number of different turbulent flows observed in nature. The most of fluids, which one could find on the Earth, are in the turbulent state. Despite many theoretical, semi-empirical and numerical investigations of turbulence, comprehensive theory has not been framed until present time. The same applies equally to the initial stage of the onset of the turbulence, its development and transition to the stage of cascade turbulent flow. In 1985, O. M. Belotserkovskii suggested that spectrum of turbulence has been fell to independent pieces. Than the turbulence can be numerically simulated without using any sub-grid-scale model. The basic ideas of direct numerical simulation rely on the hypotheses, that large-scale vortices play a dominant role in turbulent flow and are independent from small-scale turbulence at high Reynolds numbers; molecular viscosity plays a minor role in the analysis of large-scale vortex dynamics. A lot of hydrodynamic instabilities reveal themselves through large vortex structures. Flows of this type appear in regions of large velocity gradients called the mixing zones. A simple model for a mixing zone is a shear layer.

PHYSICAL MODEL. Initial stage of the onset of vortex cascades of instability in 3D free shear flows of an ideal compressible gas, as it develops from instabilities to vortices formation to mixing, is investigated. The study is based on the following physical model: the onset of instability begins with the formation of the finite-scale structures, born due to the gradients of velocity in initial flow; the viscosity is not playing significant role in the development of instabilities; the turbulence sources energy from the large-scale structures transferring it to the small-scale structures.

NUMERICAL PROCEDURE. For simulation we used 3D Euler gas dynamics equations in Cartesian coordinates:

- Equation for the gas density: $$\frac{\partial(\rho)}{\partial t} + \mathrm{div}(\rho \mathbf{V}) = 0,$$
- Equations for thee components of velocity:

$$\frac{\partial(\rho u)}{\partial t} + \mathrm{div}(\rho u \mathbf{V}) = -\frac{\partial P}{\partial x}, \qquad \frac{\partial(\rho v)}{\partial t} + \mathrm{div}(\rho v \mathbf{V}) = -\frac{\partial P}{\partial y}, \qquad \frac{\partial(\rho w)}{\partial t} + \mathrm{div}(\rho w \mathbf{V}) = -\frac{\partial P}{\partial z},$$

- Equation for the full specific energy: $$\frac{\partial(\rho E)}{\partial t} + \mathrm{div}((\rho E + P)\mathbf{V}) = 0,$$

- Here t – the time, (x, y, z) – coordinates; $\mathbf{V} = (u, v, w)$ – components of the gas velocity along X,Y and Z-direction; ρ – the gas density; P-pressure; $E = e + \mathbf{V}^2/2$ – the full specific energy; *e – the specific internal energy*.

- The ideal gas law is used in the following form: $P = (\gamma - 1)\rho e,$

γ is the ratio of specific heats.

We used monotonic dissipative stable finite-difference schemes with positive operators. This technique requires neither special filtering nor semi-empirical modeling to approximate sub-grid turbulence. The scheme has second order of the accuracy. It doesn't use neither artificial viscosity, nor smoothing. The scheme has internal non-linear dissipative mechanism that insures damping of the short wave harmonicas. The numerical simulation of free shear flow was performed on several grids: 50x50x50, 100x100x100 and 200x200x200.

RESULTS. Let is consider the flow of matter in the integration domain (fig.1). The integration domain is shaped as 3D parallelepiped in Cartesian coordinates XYZ. We investigate the evolution of the structure of shear flow of width H = 1 in case when the initial velocity of flow along Y coordinate linearly changes from -V to V inside a shear layer. Boundary conditions along X and Y coordinates are periodic, along Z coordinate we used conditions of impermeability. Development of the cascade of instabilities is initiated by pre-set random or harmonic
disturbances of initial velocity. We used different initial conditions for the velocity :
it has random disturbance in Z-direction (amplitude is varying from 2% to 20 % of V)
inside the shear layer and it has regular disturbance in X- and Z- directions:

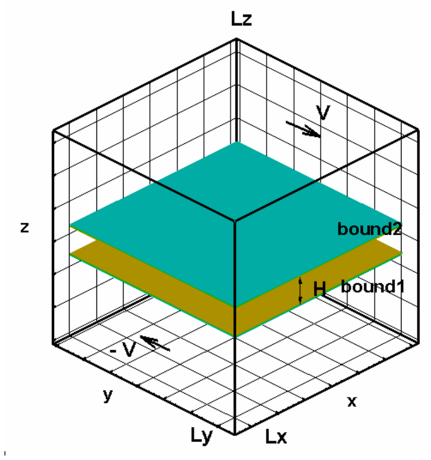

Figure 1. Integration domain, boundary and initial conditions.

$$u = \sum_i ampl[i] * \sin(a[i]x) * \cos(b[i](x - c[i])),$$

$$w = \sum_i ampl[i] * \sin(a[i]y) * \cos(b[i](y - c[i]));$$

Number of mod of harmonic disturbances is varying from 1 to 15.
We numerically study the influence on the vortex cascade of instabilities provided by change in initial conditions, dimensions of the integration domain and the width of the shear layer.
1. Let us analyze the influence on the shear flow provided by random disturbances of initial velocity of *different amplitude* (fig 2).

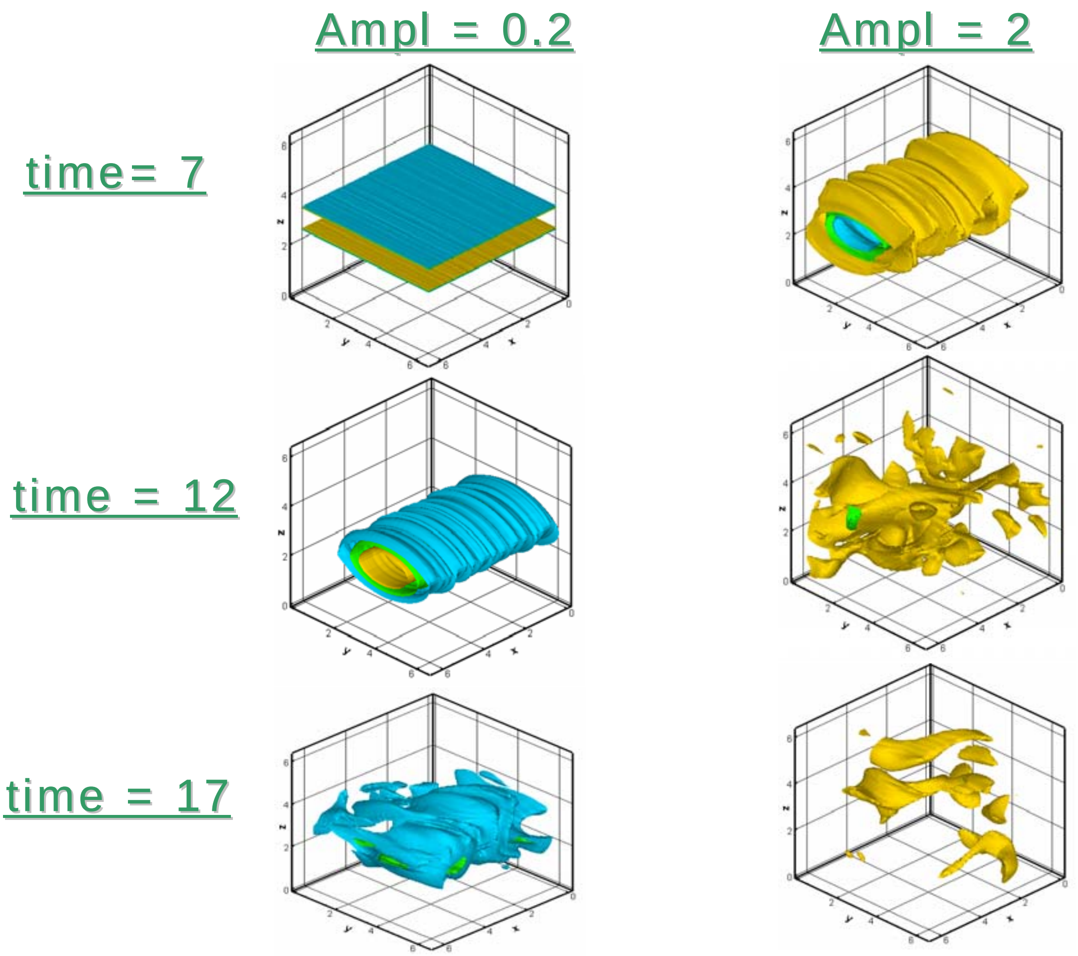

Figure 2. Equiscalar surfaces of the specific concentration depending on the amplitude of disturbance.

The evolution of the flow with different amplitudes of initial velocity is following: the turbulent flow with the large amplitude ampl = 2 is formed in double-quick time in comparison with the flow with small amplitude ampl = 0.2. By the time moment t=7 flow with the large amplitude is already in the form of large-scale vortex. At the same time, the flow with small amplitude does not reached any phase of instability. By the time moment t=12 first flow is already turbulent, second flow is in the form of large-scale vortex. By the time moment t=17 both flows are turbulent.

2. Let us analyze the influence on the shear flow provided by the *size of integration domain*(fig.3).

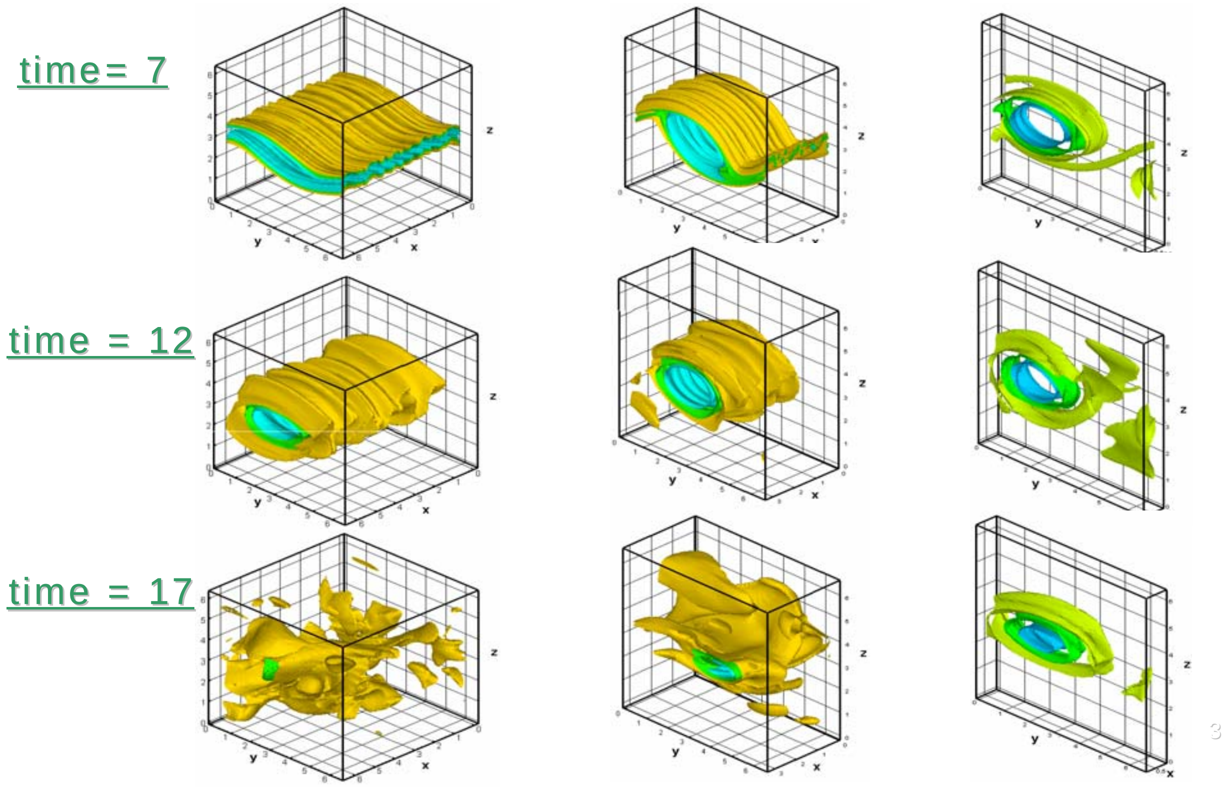

Figure 3. Equiscalar surfaces of the specific concentration depending on the length of the shear flow along x-direction in case of random initial velocity disturbance.

Length of shear layer is decreased from 2 π and till π/4 .The evolution of the flow *at the beginning* demonstrates quasi-two-dimensional nature for all shear layer sizes Lx (the onset of instability begins with the formation of large scale vortices). However, evolving further, the vortex remains quasi-two-dimensional only in case of $Lx / Ly << 1$. In other cases the vortex is unstable. The structure of the vortex cascade is transformed into turbulence area for the *large width* of shear layer Lx. The large-scale vortex changes its shape in time and finally gets destroyed. For the *small width* of shear layer Lx the vortex flow is formed within the same timeframe as in the former case, however after the formation it remains stable for any period of calculations. Critical sizes for the turbulent flow are $\pi/4<Lx<2\pi$; critical sizes for the quasi-two-dimensional case are $0<Lx<\pi/4$.

We obtained the same pattern of turbulent flow in case of harmonic initial velocity disturbance (fig.4). Similarly to the case when the initial disturbance was random, as a result of the simulation we have obtained a single large vortex. Likewise, the stability of the structure is dependent on the size of the area along X direction.

Critical size of the turbulent flow is also alike: at $\pi/4<Lx<2\pi$ the vortex is decomposing with time and the flow becomes unstable. At $0<Lx<\pi/4$ the large vortex is stable in time.

This behavior is not changing when different harmonic disturbances are applied; neither number of modes, nor amplitude nor frequency and shift of the harmonic disturbance do not provide notable influence on the pattern of the flow.
By this means, we can initiate turbulent flow pattern using regular initial condition for the velocity. This proves that the existence of the vortex cascade is not an accidental phenomena and can develop itself in case of regular initial disturbance of the velocity.

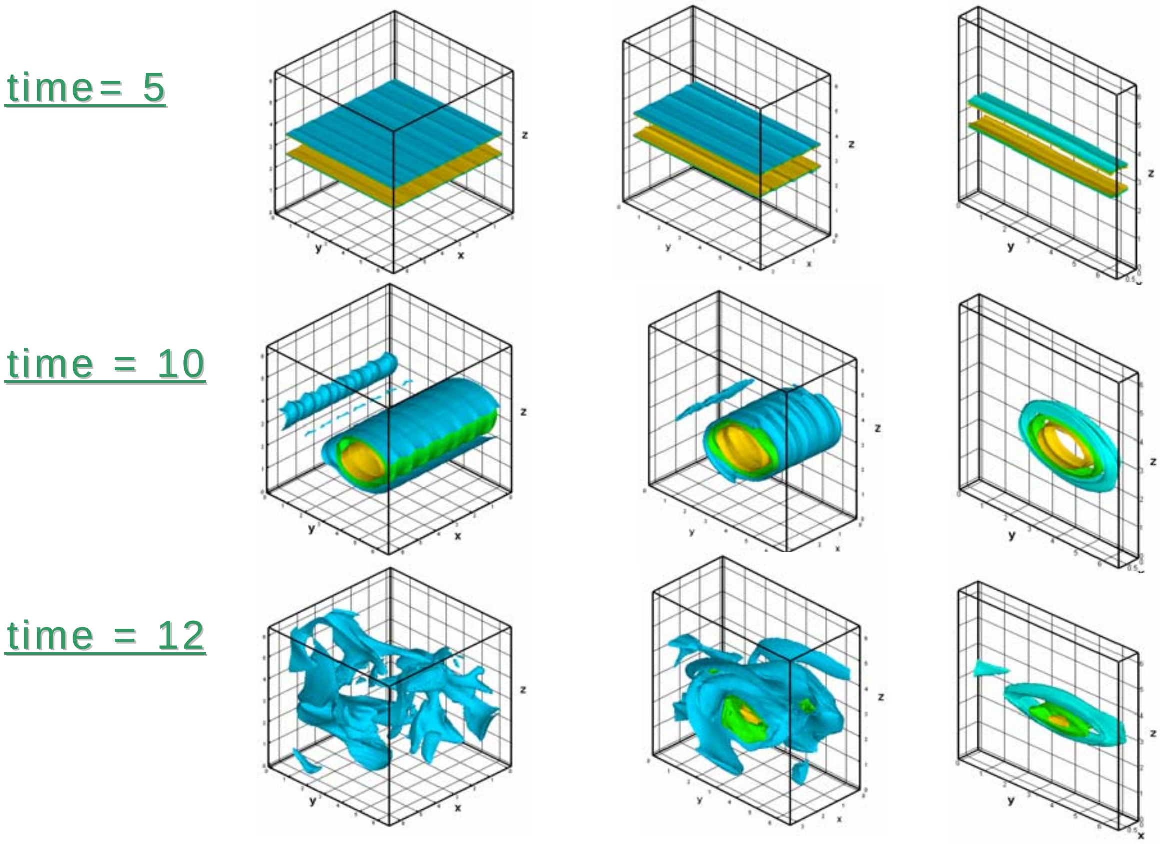

Figure 4. Equiscalar surfaces of the specific concentration depending on the length of the shear flow along x-direction in case of regular initial velocity disturbance.

3. The distribution of *the turbulent energy* E_{turb} is shown on Fig.5. At the initial stage at t=5 the energy has a clear peak in the middle of the calculation domain. As the flow develops further, at t=10 the height of the peak reaches its maximum while coordinates of the peak correspond to the center of the large vortex. This shows that large vortexes in the turbulent flow do not only define the structure of the flow but also carry most of the energy. At later phases, as the flow is transformed into a turbulent mode, the energy is gradually transferred from large structures to smaller ones and at the end finally dissipates into the heat, in correspondence with Richardson-Kolmogorov theory.

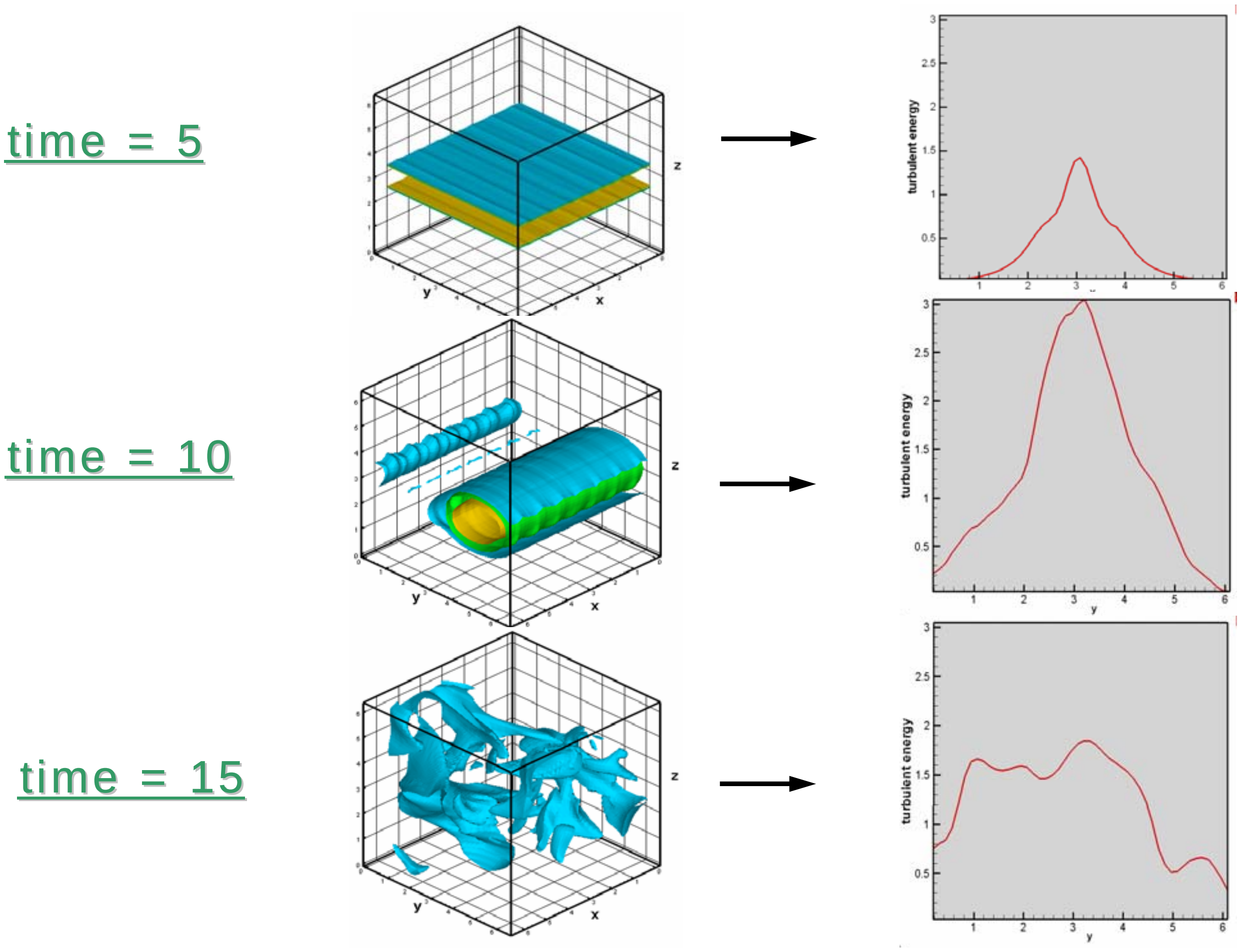

Figure 5. Equiscalar surfaces of the specific concentration and the corresponding distribution of the turbulent energy.

CONCLUSIONS. In free shear flow the birth of turbulence is connected with large vortex structures. Our numerical study of the initial phase of the turbulent flow has shown that:

- The instability is formed as a result of a decay of a vortex cascade structure ;
- We have identified conditions that lead to formation of the vortex cascade;
- We proved that harmonic initial velocity disturbance can initiate turbulent flow pattern similar to the case when disturbance is random.

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