One-point structure tensors in Rayleigh-Taylor turbulence

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Outline

Motivation

Motivation

Linear phases (RDT) in RT with complex acceleration history

Acceleration modification during RT instability changes automatically the Froude number defined as:

$$\mathcal{F}_r = \frac{\varepsilon}{kN} \text{ with } N = \left| \frac{g}{\overline{\rho}} \frac{\partial \overline{\rho}}{\partial z} \right|^{1/2}. \qquad (1)$$

When $\mathcal{F}_r \ll 1$ we have a "RDT" phase (Rapid Distorsion theory) *i.e.* non-linear terms are negligeable compared to the buoyancy terms in momentum/concentration equations for turbulent fluctuations.

The classical linear RDT equations for unstable, homogeneous, low Atwood, incompressible flow write in Fourier space:

$$(\partial_t - \nu\kappa^2)\hat{u}_i(\vec{\kappa}, t) = -\left(\delta_{i3} - \frac{\kappa_i\kappa_3}{\kappa^2}\right)\hat{\rho}(\vec{\kappa}, t), \qquad (2)$$

$$(\partial_t - \lambda\kappa^2)\hat{\rho}(\vec{\kappa}, t) = -N^2\hat{u}_3(\vec{\kappa}, t). \qquad (3)$$

cf. A.Townsend (1976), H.Hanazaki et al. (1995)

Motivation

Difficulty of modeling linear phases for single point turbulent models

One-point turbulent models (based on R_{ij} for instance) are deficient when capturing RDT phases. Indeed,

- rapid pressure correlation terms are strongly dependent on spectral wave number directionality;
- the characteristics of turbulence usually change a lot during RDT phases;
- classical one-point turbulent models lack informations. For instance, R_{ij} gives "componentality" but says nothing about "dimensionality" of turbulence.

cf. Kassinos et al., JFM 428 (2001)

Analytic solutions for RDT equations exist and can be compared to classical models.

For example, assuming that Fourier expressions $\hat{R}_{ij}$, $\hat{\rho}\hat{u}_3$, $\hat{\rho}\hat{\rho}$ depends on κ only (and not $\vec{\kappa}$) one obtains a simple closure:

$$\frac{d < u_3 u_3 >}{dt} = -\frac{4}{3} < \rho u_3 >, \tag{4}$$

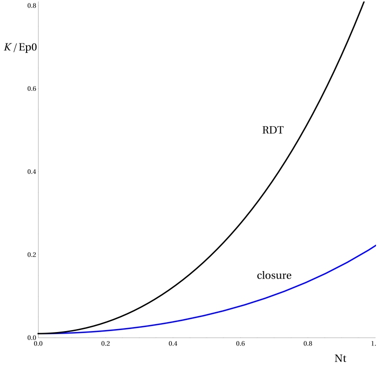
$$\frac{d < \rho u_3 >}{dt} = -N^2 < u_3 u_3 > -\frac{2}{3} < \rho\rho >, \tag{5}$$

$$\frac{d < \rho\rho >}{dt} = -2N^2 < \rho u_3 > . \tag{6}$$

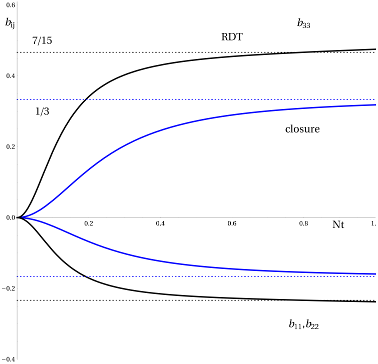
Motivation

Example of linear phase in homogeneous unstable stratification

Time evolution for turbulent kinetic energy:



Anisotropy evolution for Reynolds tensor:



As expected, very bad agreement between model and theory.

The objective is

1. Study RT during RDT phases with DNS
2. Define new tensors to take into account dimensionality of $< \rho\rho >$
3. Propose a closure for linear phase using the new tensors

DNS of RT during linear phase

DNS of RDT RT phase

TRICLADE code (cf M.Boulet and J.Griffond 10th IWPCTM)

Solves 3D compressible Navier-Stokes+concentration equations for mixtures of perfect gases.

Presently used scheme (hyperbolic part):

- high (5th) order one-step scheme with uniform time and space accuracy;

  cf. V. Daru and C. Tenaud, JCP 193 (2004)

- directional splitting;
- direct Euler solver (not Lagrange+projection);
- wave propagation method;

  cf. R.J LeVêque

- different Riemann solvers.

  cf. E.F. Toro

Presently used scheme (elliptic part+sources): operator splitting, 2nd order treatment for viscous-diffusive terms and for sources.

DNS of RDT RT phase

Simulation characteristics

- mesh $512 \times 256 \times 256$, $H = 1m$;
- Atwood number $At = 0.1$, viscosity $\nu = 10^{-5} m^2 s^{-1}$, Schmidt number $Sc = 1$ and Prandtl number $Pr \approx 3/4$;
- initial stratification adiabatic with $\gamma = 5/3$, pressure $p_0 \approx 4 m^2 s^{-2}$ (incompressible limit);
- Initial perturbations for RT instability " M256 type" ;

  cf. G.Dimonte et al., Phys. Fluids 16 (2004)

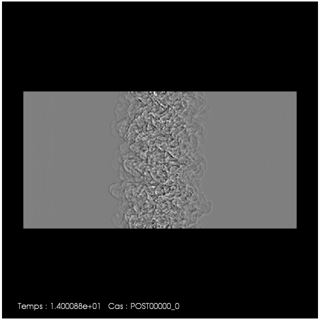
- Gravity: $G = 0.2 ms^{-2}$ for $T \leq 14s$ before RDT phase, then $G = 20 ms^{-2}$ for $T > 14s$;
- Rescaling of mean pressure at the beginning of RDT phase to avoid shock creation.

  cf. A.Llor and D. Youngs IWPCTM (2002)

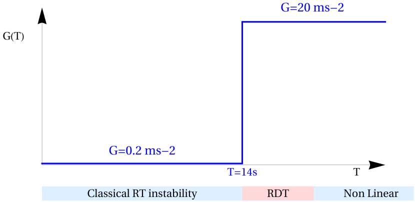
DNS of RDT RT phase

Global results

Shadowscopy:



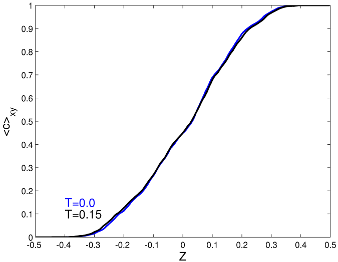
Time evolution of acceleration:



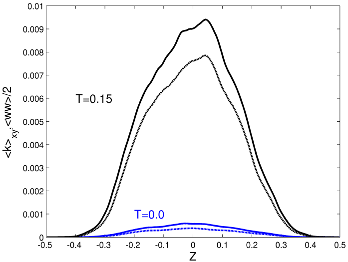
DNS of RDT RT phase

Determination of linear phase

Mean concentration profile:



Mean kinetic energy profile:



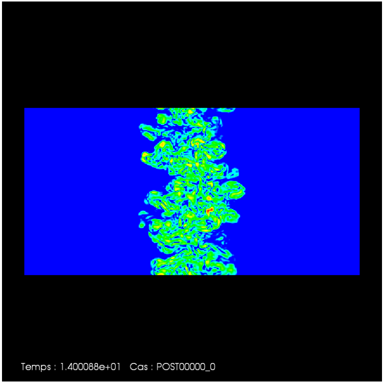
Linear phase (observed), between $T = 14s$ and $T = 14.15s$:

- mixing zone length is still, $N = 2.7s^{-1}$;
- turbulent kinetic energy explodes.

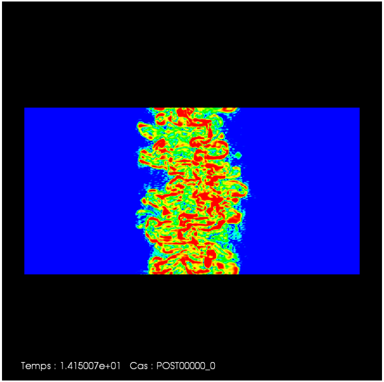
DNS of RDT RT phase

Determination of linear phase

Vorticity at beginning RDT phase:



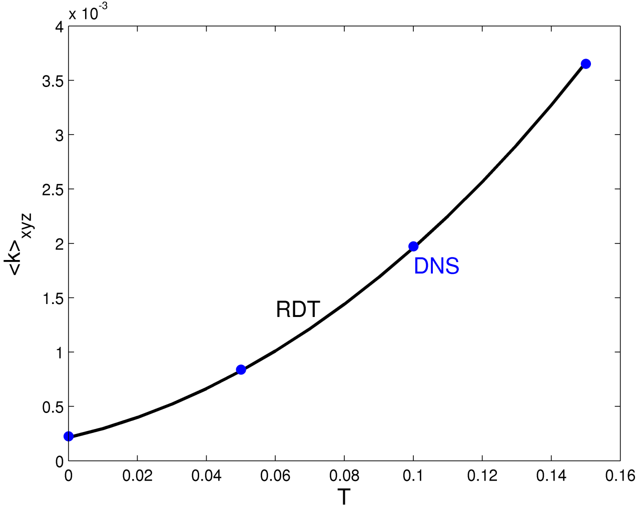
Vorticity at the end of RDT phase:



DNS of RDT RT phase

Validity of RDT equations

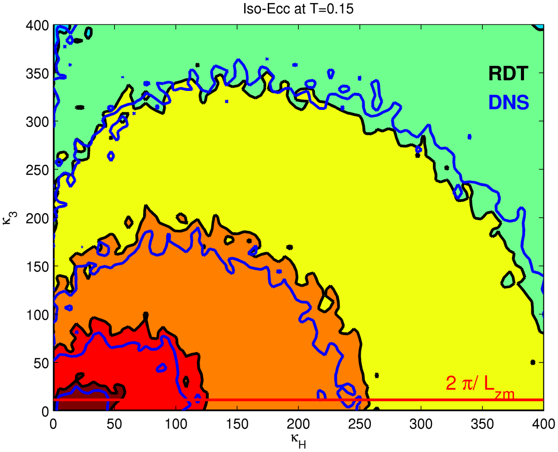
Total kinetic energy, comparison DNS/RDT:



DNS of RDT RT phase

Validity of RDT equations

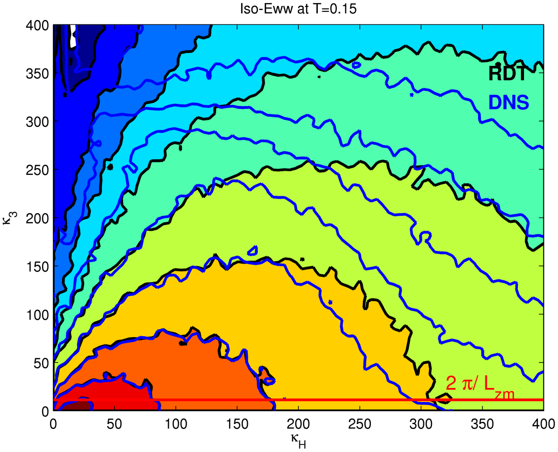
Spectral structure of density spectra, comparison DNS/RDT:



DNS of RDT RT phase

Validity of RDT equations

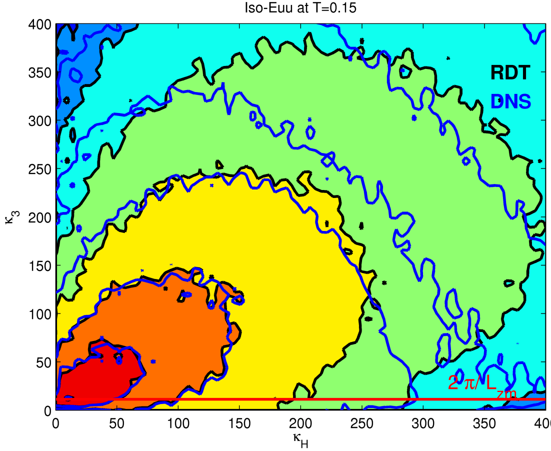
Spectral structure of vertical velocity, comparison DNS/RDT:



DNS of RDT RT phase

Validity of RDT equations

Spectral structure of horizontal velocity, comparison DNS/RDT:

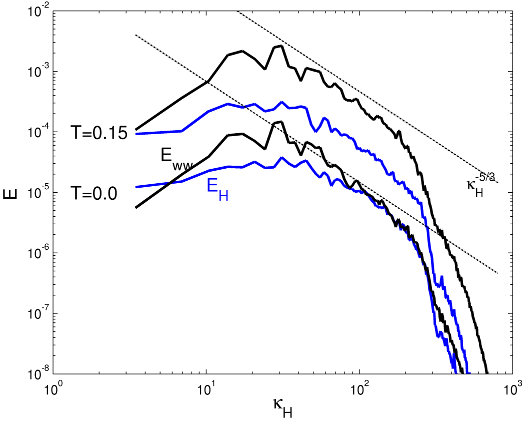


Good agreement at large scale where energy is contained

DNS of RDT RT phase

Spectra

Velocity spectra at center of mixing zone:

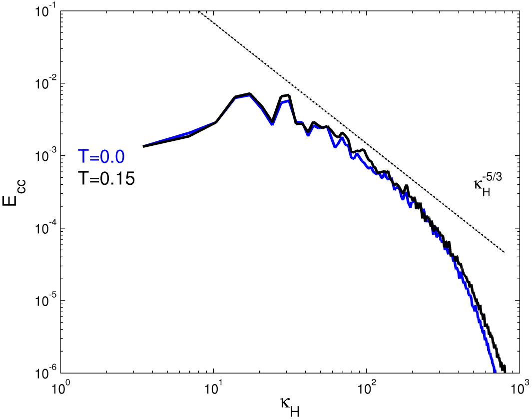


Growth of anisotropy.

DNS of RDT RT phase

Spectra

Concentration spectra:

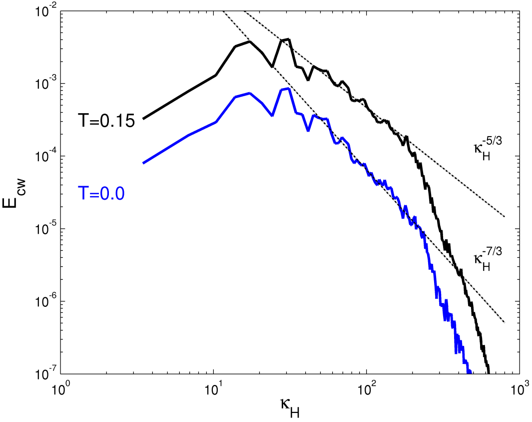


Small modification of concentration field.

DNS of RDT RT phase

Spectra

Mass flux spectra:



Slope modification observed? *cf.* J.L. Lumley, Phys. Fluids 10 (1967)

One-point structure tensors in Rayleigh-Taylor during linear RDT phases

One-point hydrodynamic tensors are defined by:

$$\text{Componentality } R_{ij} = \epsilon_{ipq}\epsilon_{jts} < \psi_{q,p}\psi_{s,t} >_{xy}, \quad (7)$$

$$\text{Dimensionality } D_{ij} =< \psi_{n,i}\psi_{n,j} >_{xy}, \quad (8)$$

$$\text{Circulicity } F_{ij} =< \psi_{i,n}\psi_{j,n} >_{xy}, \quad (9)$$

$$\text{"Inhomogeneity" } C_{ij} =< \psi_{i,n}\psi_{n,j} >_{xy} . \quad (10)$$

where the stream vector $\vec{\psi}$ is solution of

$$\psi_{i,nn} = -\omega_i, \quad \psi_{i,i} = 0, \quad u_i = \epsilon_{its}\psi_{s,t}. \quad (11)$$

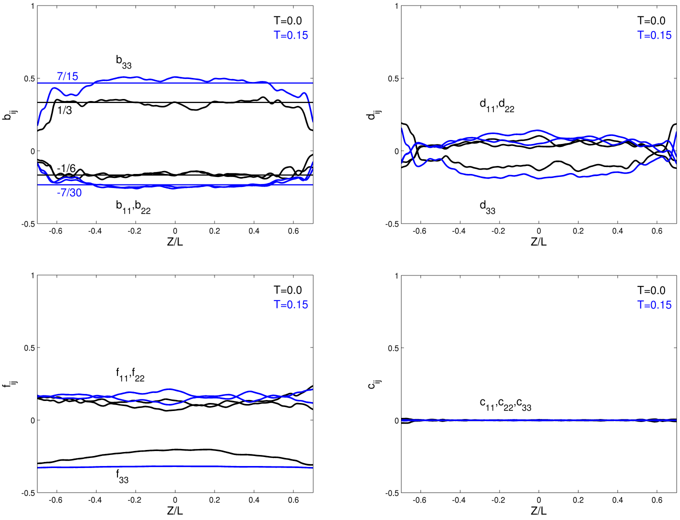
cf. Kassinos et al., JFM (2001) also C.Cambon et al. Phys. Fluids 4 (1992).

They are not independent as,

$$R_{ij} + D_{ij} + F_{ij} - (C_{ij} + C_{ji}) = 2k^2\delta_{ij}. \quad (12)$$

One-point structure tensors

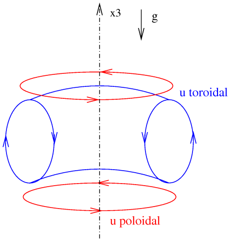
Evolution of purely kinematic tensors



"Inhomogeneity" tensor is equal to 0:

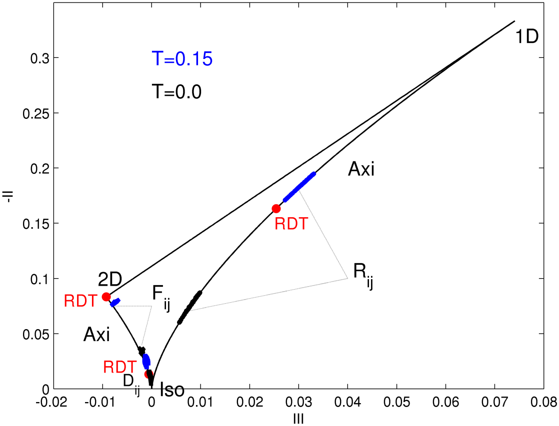
Poloidal velocity correlations negligeable.

$$\vec{u} = \vec{\nabla} \times \underbrace{s_{pol}\mathbf{x_3}}_{(0,0,\psi_3)} + \vec{\nabla} \times \vec{\nabla} \times \underbrace{s_{tor}\mathbf{x_3}}_{(\psi_1,\psi_2,0)} \quad (13)$$



One-point structure tensors

Representation of purely kinematic tensors in Lumley triangle



$$II(A) = -\frac{1}{2} Trace(A^2), \; III(A) = \frac{1}{3} Trace(A^3) \quad (14)$$

One-point structure tensors

Definition of new velocity/tensor

We define an enhanced velocity by adding a solenoidal part dependent on density ρ:

$$\hat{v}_i(\vec{\kappa}, t) = \hat{u}_i(\vec{\kappa}, t) + \hat{u}_i^{(\rho)}(\vec{\kappa}, t), \text{ with } \hat{u}_i^{(\rho)}(\vec{\kappa}, t) = \frac{\kappa_i}{\kappa N}\hat{\rho}(\vec{\kappa}, t). \tag{15}$$

cf. J.J. Riley and al (1981), F.S. Godeferd and C. Cambon Phys Fluid 6 (1994)

The time evolution in RDT is determined by the linear equation:

$$\frac{d}{dt}\hat{v}_i = L_{ij}\hat{v}_j, \tag{16}$$

with

$$L_{ij} = -N\left(\mathcal{P}_{i3}\frac{\kappa_j}{\kappa} + \mathcal{P}_{j3}\frac{\kappa_i}{\kappa}\right), \text{ with } \mathcal{P}_{ij} = \delta_{ij} - \frac{\kappa_i\kappa_j}{\kappa^2}. \tag{17}$$

The second order spectral tensor can be formed from enhanced velocity:

$$\hat{V}_{ij}(\vec{\kappa}) =< \hat{v}_i(\vec{\lambda})\hat{v}_j^*(\vec{\kappa} - \vec{\lambda}) > \tag{18}$$

$$\frac{d}{dt}\hat{V}_{ij} = L_{in}\hat{V}_{nj} + L_{jm}\hat{V}_{mi}^* \tag{19}$$

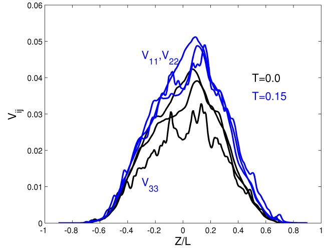
Dimensionality for ρ defined as:

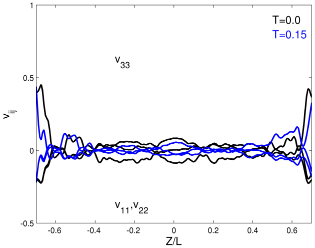
$$\hat{D}_{ij}^{(\rho)}(\vec{\kappa}) =< u_i^{(\rho)}(\vec{\lambda})u_j^{(\rho)*}(\vec{\kappa} - \vec{\lambda}) > \tag{20}$$

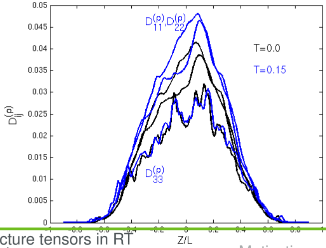
One-point structure tensors

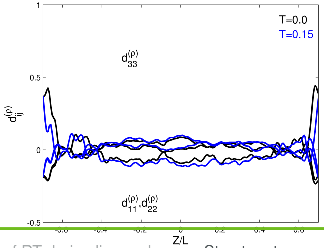
evolutions of new tensor in physical space

Integration over all wave numbers:



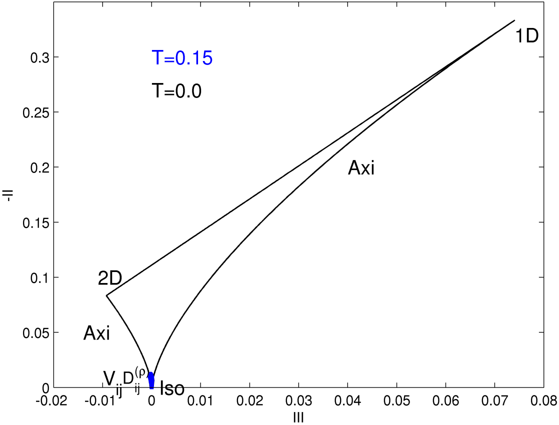






One-point structure tensors

Representation of mixed tensors in Lumley triangle



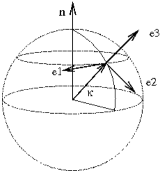
cf. different from dimensionality tensor X.Albets-Chico and al. IUTAM (2010).

Anisotropy decomposition in linear RT

Representation of the enhanced velocity tensor in a Craya-Herring frame

The Craya-Herring frame is defined by,

$\mathbf{e^1} = \vec{\kappa} \times \mathbf{n}/|\vec{\kappa} \times \mathbf{n}|, \ \mathbf{e^2} = \vec{\kappa} \times \mathbf{e^1}/|\vec{\kappa} \times \mathbf{e^1}|, \ \mathbf{e^3} = \vec{\kappa}/|\vec{\kappa}|.$



We have for axisymetric turbulence,

$$\frac{1}{2}\mathbf{e^i}_n \hat{V}_{nm}\mathbf{e^j}_m = \begin{pmatrix} \Phi_1 & 0 & 0 \\ 0 & \Phi_2 & \Psi^* \\ 0 & \Psi & \Phi_3 \end{pmatrix} \qquad (21)$$

where

- Φ_1 represents turbulent kinetic energy of poloidal velocity;
- Φ_2 is the turbulent kinetic energy of toroidal velocity;
- Φ_3 define a potential energy from the density spectrum;
- Ψ generates the vertical buoyancy flux.

$$\frac{1}{2} < u_3\rho >= N \int \Psi_R sin(\theta) d^3\vec{\kappa} \qquad (22)$$

Anisotropy decomposition

Expansion for the tensors

The evolution equations write:

$$\frac{\partial \Phi_1}{\partial t} = 0, \quad \frac{\partial \Psi_I}{\partial t} = 0, \tag{23}$$

$$\frac{\partial \Phi_2}{\partial t} - N sin(\theta) \Psi_R = 0, \tag{24}$$

$$\frac{\partial \Phi_3}{\partial t} - N sin(\theta) \Psi_R = 0, \tag{25}$$

$$\frac{\partial \Psi_R}{\partial t} - 2N sin(\theta)(\Phi_2 + \Phi_3) = 0. \tag{26}$$

This suggests to use the decomposition:

$$\frac{\Phi_2(\vec{\kappa}, t) + \Phi_3(\vec{\kappa}, t)}{2} = H_0^{(dir)}(\kappa, t) + H_1^{(dir)}(\kappa, t) \sin^2\theta + \ldots + H_n^{(dir)}(\kappa, t) \sin^{2n}\theta + \ldots \tag{27}$$

$$\Psi_R(\vec{\kappa}, t) = \sin\theta \left(H_0^{(pola)}(\kappa, t) + H_1^{(pola)}(\kappa, t) \sin^2\theta + \ldots + H_n^{(pola)}(\kappa, t) \sin^{2n}\theta + \ldots \right) \tag{28}$$

Compatibility with constraint of polar isotropy, at $\kappa_1 = \kappa_2 = 0$, $\Psi = 0$ and $\Phi_1 = \Phi_2$.

$$\frac{\partial H_n^{pola}}{\partial t} = 2N H_n^{dir}, \quad \frac{\partial H_{n+1}^{dir}}{\partial t} = N H_n^{pola}. \tag{29}$$

Anisotropy decomposition

Expansion for the tensors

Assuming to simplify that poloidal velocity equals 0 ($\Phi_1 = 0$) we have:

$$\frac{1}{2}R(t) = \underbrace{\int \frac{\Phi_2 - \Phi_3}{2}(\vec{\kappa})M^{(u)}(\vec{\kappa})d^3\vec{\kappa}}_{R^{polarized}} + \underbrace{\int \frac{\Phi_2 + \Phi_3}{2}(\vec{\kappa}, t)M^{(u)}(\vec{\kappa})d^3\vec{\kappa}}_{R^{directional}(t)} \quad (30)$$

$$= R^{polarized} + \sum_n R^{(n)} \int H_n^{(dir)} d\kappa(t) \quad (31)$$

in the same way,

$$\frac{1}{2}D_{(\rho)}(t) = D_{(\rho)}^{polarized} + \sum_n D^{(n)} \int H_n^{(dir)} d\kappa(t) \quad (32)$$

with

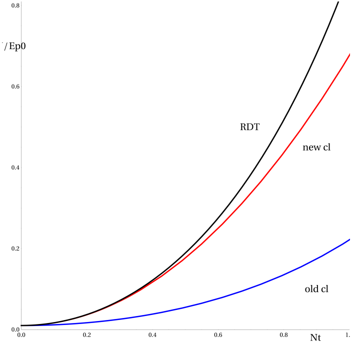
$$R^{(n)} = \int \kappa_H^n M^{(u)} d^3\vec{\kappa}, \; D^{(n)} = \int \kappa_H^n M^{(\rho)} \, d^3\vec{\kappa}. \quad (33)$$

$$M^{(u)}(\vec{\kappa}) = \begin{pmatrix} \frac{\kappa_1^2\kappa_3^2}{\kappa_H^2\kappa^2} & \frac{\kappa_1\kappa_2\kappa_3^2}{\kappa_H^2\kappa^2} & -\frac{\kappa_1\kappa_3}{\kappa^2} \\ \frac{\kappa_1\kappa_2\kappa_3^2}{\kappa_H^2\kappa^2} & \frac{\kappa_2^2\kappa_3^2}{\kappa_H^2\kappa^2} & -\frac{\kappa_2\kappa_3}{\kappa^2} \\ -\frac{\kappa_1\kappa_3}{\kappa^2} & -\frac{\kappa_2\kappa_3}{\kappa^2} & \frac{\kappa_H^2}{\kappa^2} \end{pmatrix}, M^{(\rho)}(\vec{\kappa}) = \begin{pmatrix} \frac{\kappa_1^2}{\kappa^2} & \frac{\kappa_1\kappa_2}{\kappa^2} & \frac{\kappa_1\kappa_3}{\kappa^2} \\ \frac{\kappa_1\kappa_2}{\kappa^2} & \frac{\kappa_2^2}{\kappa^2} & \frac{\kappa_2\kappa_3}{\kappa^2} \\ \frac{\kappa_1\kappa_3}{\kappa^2} & \frac{\kappa_2\kappa_3}{\kappa^2} & \frac{\kappa_3^2}{\kappa^2} \end{pmatrix} d^3\vec{\kappa}, \quad (34)$$

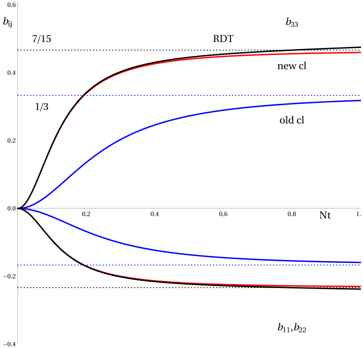
Model for RDT using new tensor

Back to the first example!

Time evolution for turbulent kinetic energy:



Anisotropy evolution for Reynolds tensor:



Improvement due to better modeling of anisotropy evolution.

Conclusion

- DNS of linear RDT phase for Rayleigh-Taylor instability have been performed.
- An analysis by one-point structure tensors has been presented.
- An original decomposition of anisotropy has been realized based on Craya-Herring representation of enhanced velocity.
- This allows a new one-point closure for linear RT phase based on dimensionality of $< \rho\rho >$.

Thank you for your attention