The Numerical Studies on Rayleigh-Taylor Instability of Aluminum Plates Driven by Detonation

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Contents

Introduction

- The RT instability can occur in solids that are accelerated by a lower-density fluid.
- The material strength plays a role in stabilizing or reducing the perturbation growth in the solid state.
- There is no clear understanding of the RT instability processes in real media possessing strength, compressibility, viscosity, or phase transitions.

Introduction

- The published literature contains contradictory conclusion on the effect of the principal factors affecting instability evolution, including
  - wavelength
  - perturbation amplitude
  - material strength properties
  - loading history

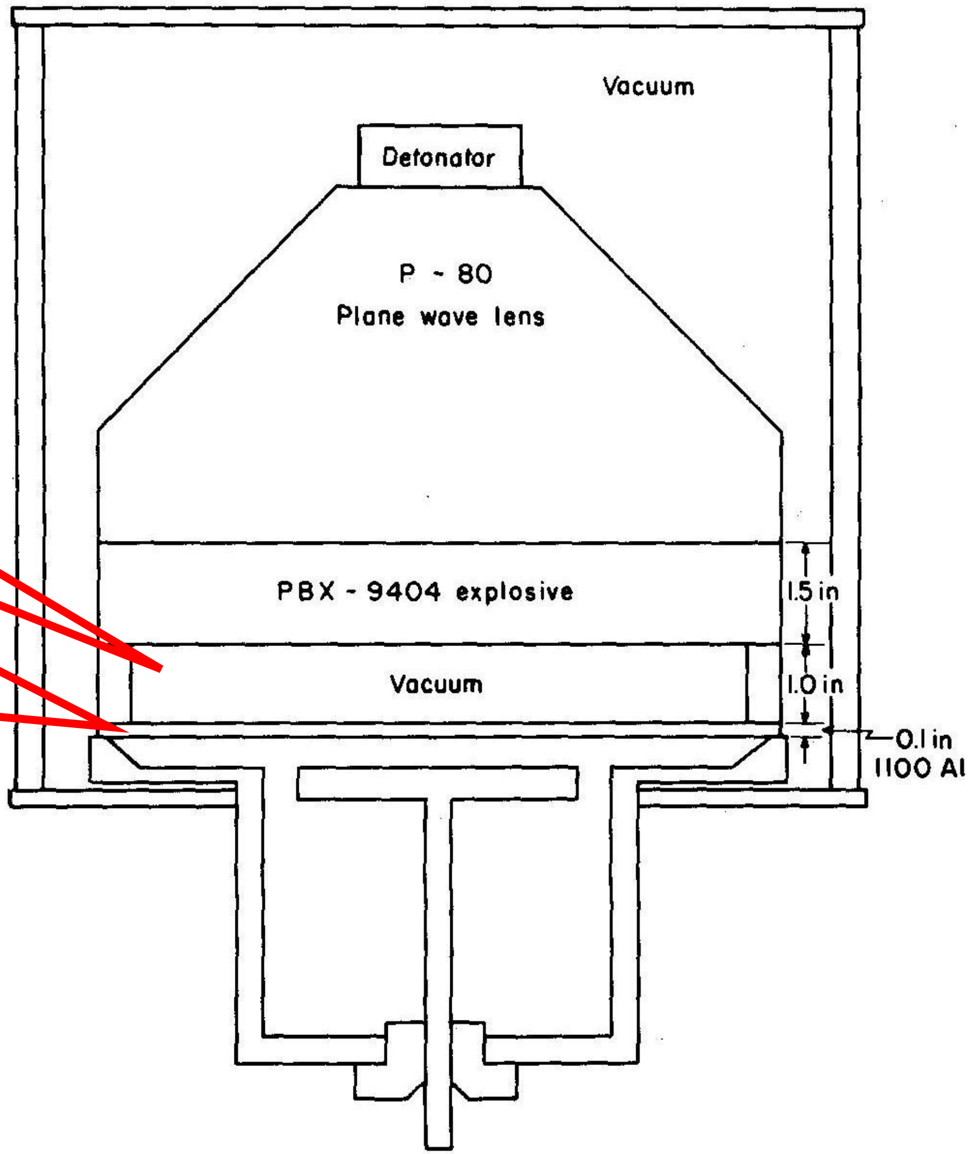
Introduction

- Using elastic-plastic hydrodynamic code (Compatible Hydrodynamics Analysis Program, CHAP)
- Explore the Rayleigh-Taylor instability of Al plates driven by HE detonation
- Both equation of state (EOS) and constitutive model

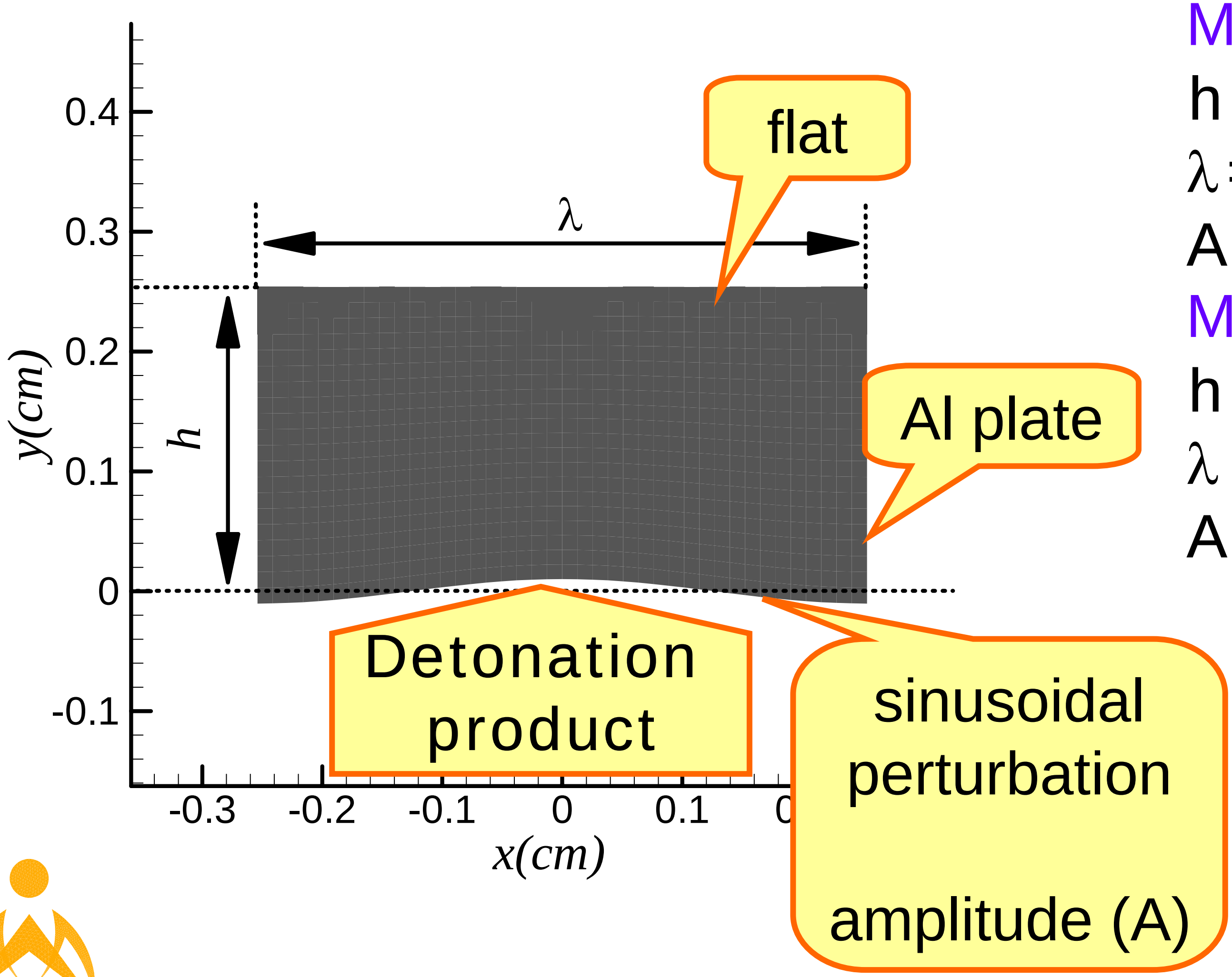
Barnes experiment

The gaseous HE detonation products expanding across a void.

The sinusoidal perturbations were meachined.



Model



Model one：
h=2.54mm，
λ=5.08mm，
A=0.102mm；
Model two ：
h=2.54mm，
λ =2.54mm，
A=0.050mm。

NOTE：
h=const
A/λ=const

Numerical method

- CHAP is a general purpose elastic-plastic hydrodynamic code.
  - Compatible Lagrangian numerical method.
  - Explicit time integration: predictor-corrector
  - Hourglass: subzonal pressure
  - Shock wave: tensor artificial viscosity
  - Contact-impact algorithm
  - Elastic-plastic effect
  - High-explosive (HE) reaction

Calculated approach

- Density of Al is 2.78
- Gruneisen EOS

$$p = \frac{\rho_0 c^2 \mu \left[1 + \left(1 - \frac{\gamma_0}{2}\right)\mu - \frac{a}{2}\mu^2\right]}{\left[1 - (S_1 - 1)\mu - S_2 \frac{\mu^2}{\mu + 1} - S_3 \frac{\mu^3}{(\mu + 1)^2}\right]} + (\gamma_0 + a\mu)E$$

- Steinberg constitutive model

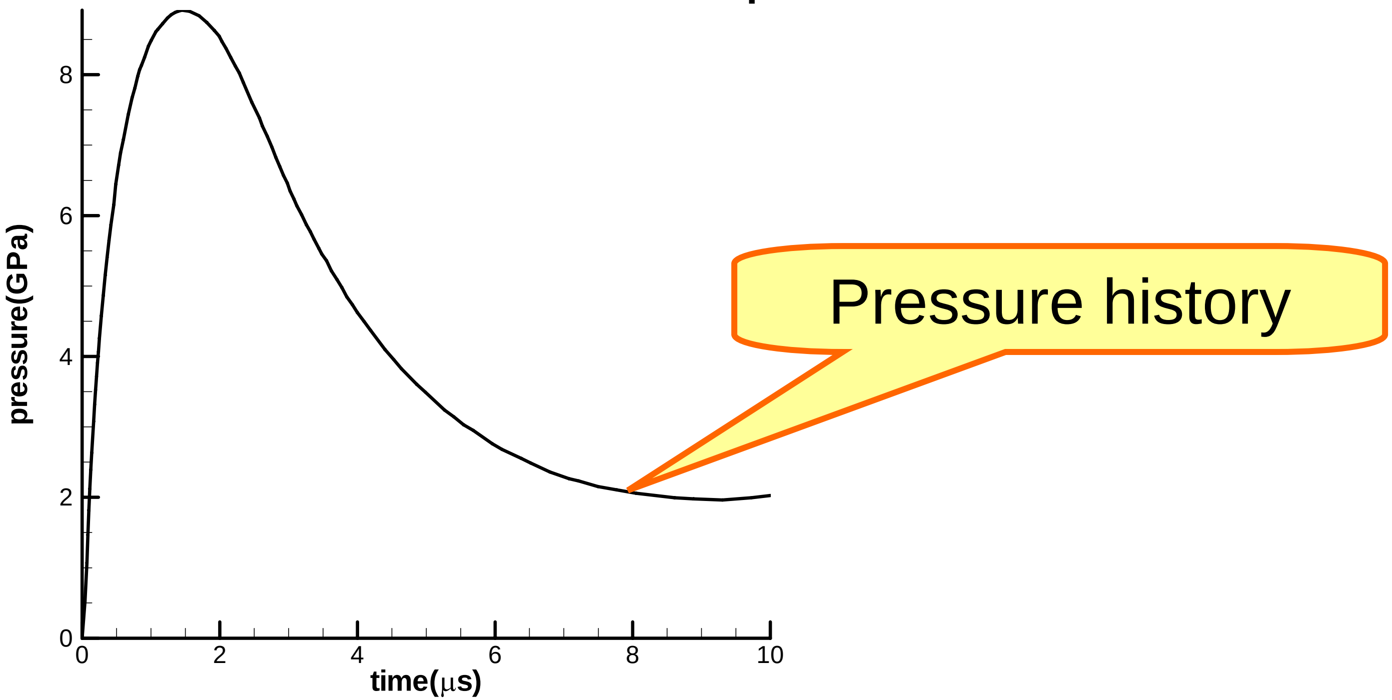
$$G = G_0 \left[1 + bpV^{1/3} - h\left(\frac{E - E_c}{3R'} - 300\right)\right] e^{-\frac{fE}{E_m - E}}$$

$$\sigma_y = \sigma_0' \left[1 + b'pV^{1/3} - h\left(\frac{E - E_c}{3R'} - 300\right)\right] e^{-\frac{fE}{E_m - E}}$$

$$\sigma_0' = \sigma_0 \left[1 + \beta(\gamma_i + \varepsilon_p)\right]^n$$

Calculated approach

HE detonation loading is treated as a time-dependent pressure drive distributed over the plate surface.



The symmetry boundaries must fall at either a maximum or a minimum of the perturbation, so the minimum lateral extent of the calculation is one-half wavelength of the perturbation.

Analysis

- Incompressible elastic material

pressure

$$\mathbf{T} = -p\mathbf{I} + \mathbf{S}$$

stress

deviatoric stress

strain

velocity

$$\mathbf{D} = \frac{1}{2}\left(\nabla\mathbf{v} + \nabla\mathbf{v}^T\right)$$

incompressibility

$$\mathbf{T} : \mathbf{D} = \mathbf{S} : \mathbf{D}$$

elastic material

$$\dot{\mathbf{S}} = 2G\mathbf{D}$$

shear modulus

Analysis

- Momentum equation $\rho\dot{\mathbf{v}} = \nabla \cdot \mathbf{T}$

$$\int \rho\dot{\mathbf{v}} \cdot \mathbf{v} d\Omega = \int \nabla \cdot \mathbf{T} \cdot \mathbf{v} d\Omega$$

Gauss theorem

$$-\oint p\mathbf{n} \cdot \mathbf{v} dS - \int \rho\dot{\mathbf{v}} \cdot \mathbf{v} d\Omega = \int \mathbf{S} : \mathbf{D} d\Omega$$

- The velocity field is given

$$\mathbf{v} = \dot{q} e^{-ky} \begin{pmatrix} \sin kx \\ \cos kx \end{pmatrix}$$

$$y = q \cos kx$$

amplitude

wavelength

$$2\pi k = \lambda$$

wavenumber

Analysis

$$-\oint p\mathbf{n}\cdot\mathbf{v}\,dS = \pi p(t)q\dot{q}$$

zero pressure boundary condition on upper surface

$$-\int \rho\dot{\mathbf{v}}\cdot\mathbf{v}\,d\Omega = -\frac{\rho\lambda\dot{q}\ddot{q}}{2k}\left(1-e^{-2kh}\right)$$

$$\int \mathbf{S}:\mathbf{D}\,d\Omega = 4\pi G\dot{q}\left(q-q_0\right)\left(1-e^{-2kh}\right)$$

$$\mathbf{S}:\mathbf{D} = 4Gk^2e^{-2ky}\dot{q}\left(q-q_0\right) \qquad \frac{d}{dt}\left(\mathbf{S}:\mathbf{Q}\right) = 4G\dot{q}ke^{-ky}$$

$$\dot{\mathbf{S}}:\mathbf{Q} = 2G\mathbf{D}:\mathbf{Q} \qquad \mathbf{D} = \dot{q}ke^{-ky}\begin{pmatrix}\cos kx & -\sin kx\\ -\sin kx & -\cos kx\end{pmatrix} = \dot{q}ke^{-ky}\mathbf{Q}$$

Analysis

- Perturbation amplitude equation for a time-dependent pressure

$$\ddot{q}+\frac{k^2}{\rho}\left(4G-\frac{p(t)}{1-e^{-2kh}}\right)q=4G\frac{k^2}{\rho}q_0$$

- Constant pressure （ $p=\rho a h$ ）
  - Cutoff wave number: $k_c=\frac{\rho a}{8G}$
  - Other theorem: $k_c=\frac{\rho a}{4G}$
- Perturbation amplitude equation for variable pressure gradient

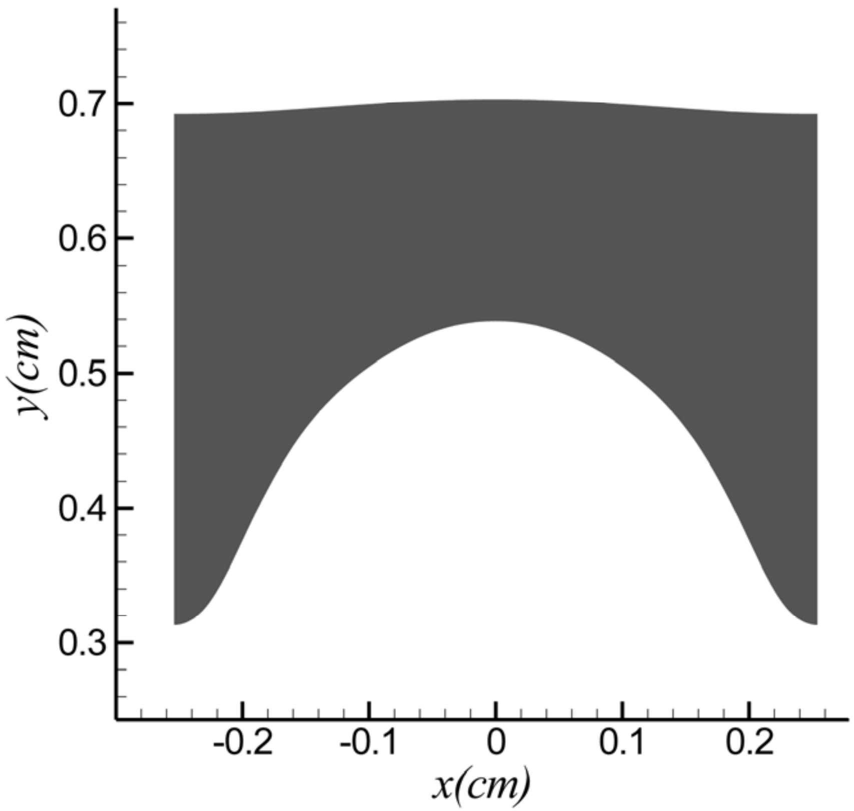
$$\ddot{q}+\frac{k^2}{\rho}\left(4G-\frac{1}{k}\frac{dP}{dy}\right)q=4G\frac{k^2}{\rho}q_0$$

Results and comments

- The perturbation growth factor histories are focused.
- The perturbation growth factor is defined that the perturbation amplitude growth is divided by initial perturbation amplitude.
- The perturbation growth factor means that the perturbation amplitude growth are multiples of initial perturbation amplitude.

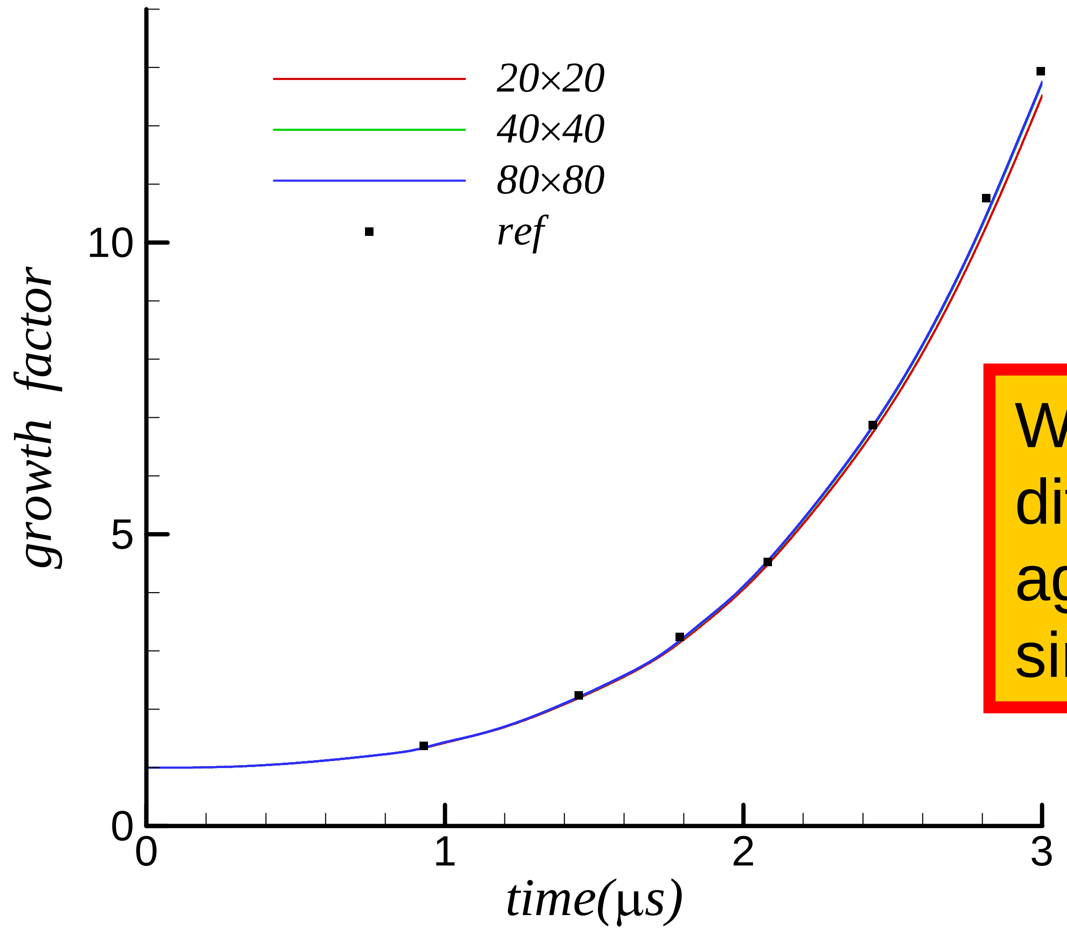
Results and comments: Case A

- For model one, assuming the material is fluid, we use only EOS for the aluminum, but without the constitutive model.



The shape of Al plate is shown in t=3.0μs.

Results and comments : Case A



When the mesh length is different, the result agrees well with Colvin simulation.

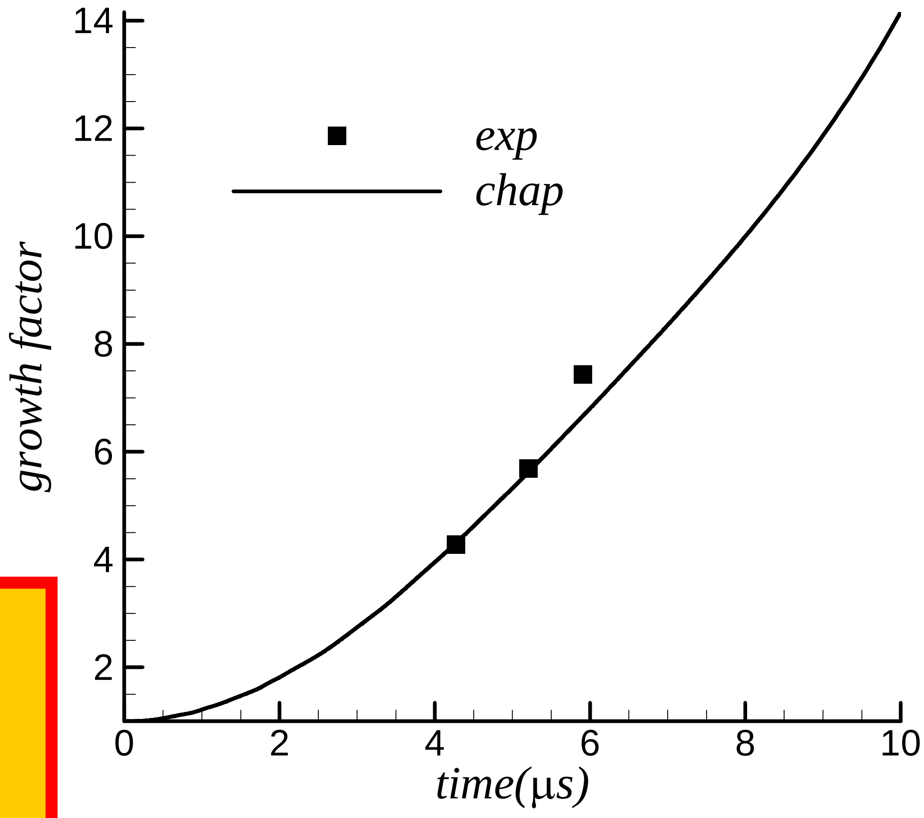
Perturbation growth factor history

Results and comments : Case B

The growth of perturbation is unstable

- For model one, we use both EOS and constitutive model.

The simulated growth matches the experiment results of Barnes..

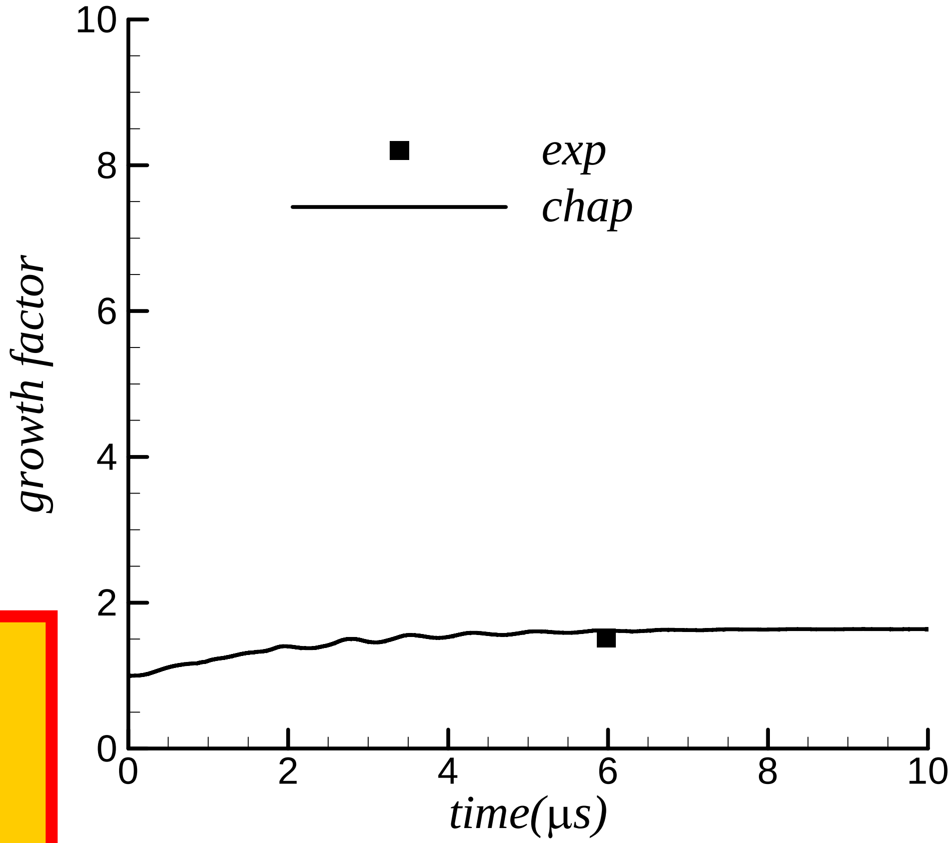


Perturbation growth factor history

Results and comments : Case C

The growth of perturbation is stable

- For model two, we use both EOS and constitutive model.



Perturbation growth factor history

The simulated growth matches the experiment results of Barnes..

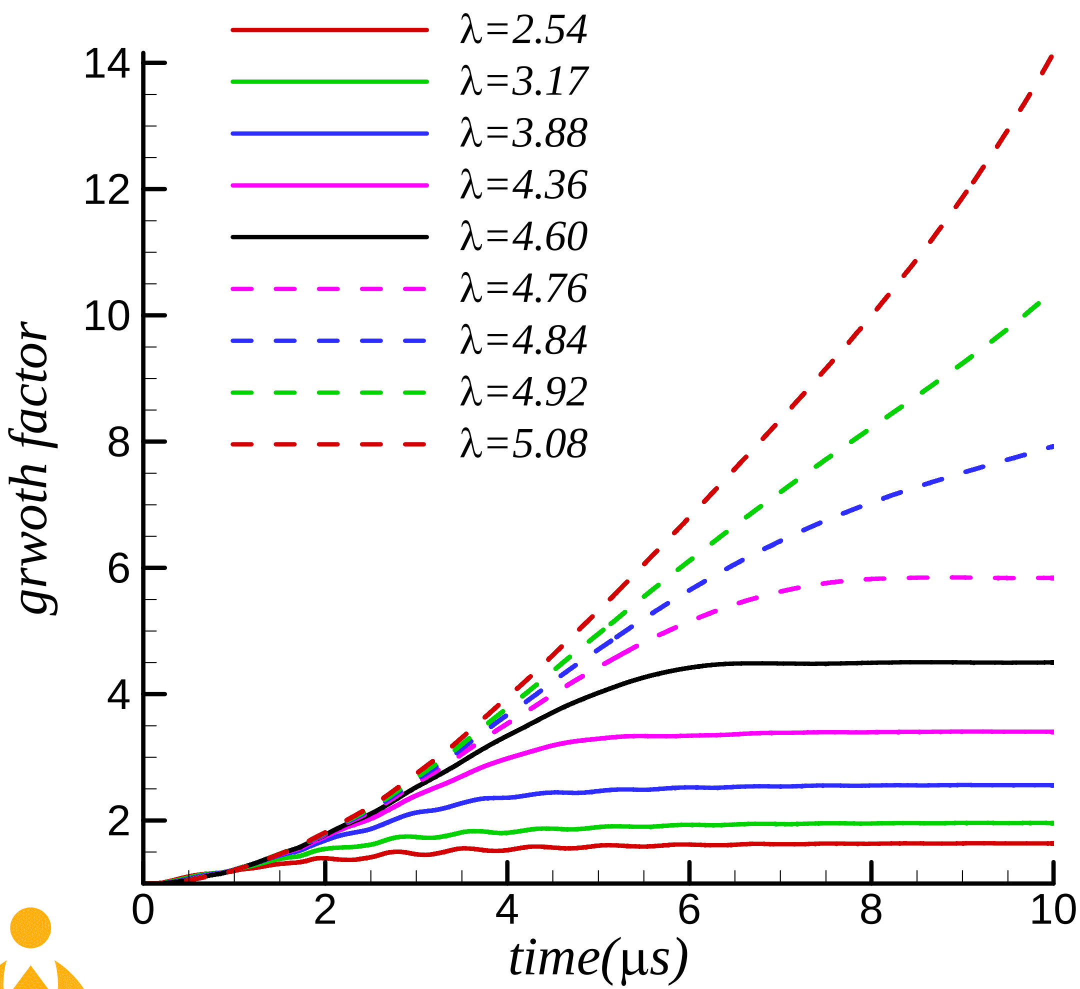
Results and comments : Case D

- The results calculated are validated.
- There is a cutoff wavelength for RT instability of the metal.
- The growth of perturbation is stable when the perturbation wavelength is smaller than the critical wavelength.
- The perturbation is stable for λ=2.54mm and the growth increases rapidly for λ=5.08mm.

Results and comments : Case D

- The question that arises for RT instability of Al plate growth is when the wavelength is from 2.54mm to 5.08mm, what is the growth factor?
- So we, using the validated CHAP, expand the numerical study with different wavelength, from 2.54mm to 5.08mm.

Results and comments : Case D



Initial wavelengths less than 4.76mm are stable, in that the perturbation doesn't develop after the growth rate is linear. The linear stage is longer as the wavelength is longer.

The perturbation growth increases rapidly for the wavelength greater than 4.84mm.

Conclusion

- The results calculated by CHAP match the experiment and simulation in the journals.
- For the numerical simulation of RT instability of the metal driven by HE detonation, the elas-tic-plastic effect must be assumed.
- The result of simulation is different with the experiment, using only EOS.

Conclusion

- The growth of perturbation agrees well with the measured growth when assuming it is elastic-plastic.
- There is a cutoff wavelength for RT instability of the metal.
- The growth of perturbation is stable for short wavelength.
- The growth increases rapidly as the wavelength increases.

Thanks