Analytic approach to nonlinear hydrodynamic instabilities driven by time-dependent accelerations

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We extend our earlier model for Rayleigh-Taylor and Richtmyer-Meshkov instabilities to the more general class of hydrodynamic instabilities driven by a time-dependent acceleration $g(t)$. Explicit analytic solutions for linear as well as nonlinear amplitudes are obtained for several $g(t)$’s by solving a Schrödinger-like equation

$$d^2 \eta / dt^2 - g(t) A \eta = 0$$

where $A$ is the Atwood number and $k$ is the wavenumber of the perturbation amplitude $\eta(t)$. In our model a simple transformation $k \rightarrow k_L$ and $A \rightarrow A_L$ connects the linear to the nonlinear amplitudes: $\eta^{\text{nonlinear}}(k, A) \sim \left(1/k_L\right) \ln \eta^{\text{linear}}(k_L, A_L)$. The model is found to be in very good agreement with direct numerical simulations. Bubble amplitudes for a variety of accelerations are seen to scale with $s$ defined by $s = \int \sqrt{g(t)} dt$, while spike amplitudes prefer scaling with displacement $\Delta x = \int [\int g(t) dt] dt$.

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