ANALYTIC APPROACH to NONLINEAR HYDRODYNAMIC INSTABILITIES DRIVEN by g(t)*

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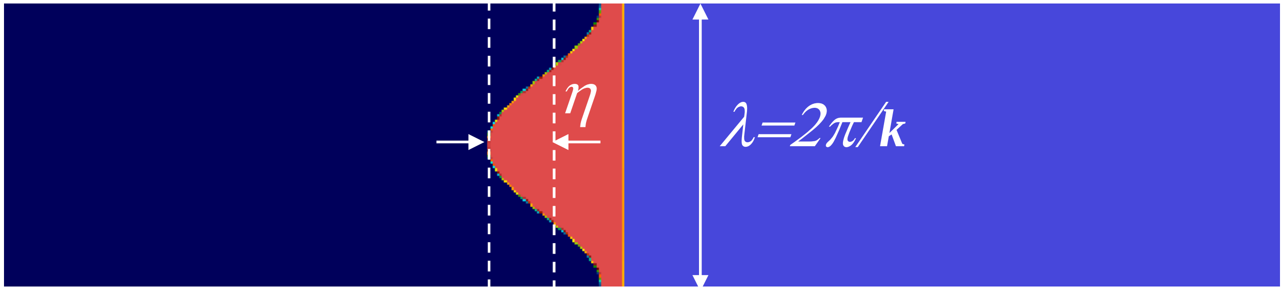

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FOUR LEVELS of COMPUTATIONAL ACCURACY

1. Euler Equations $$\frac{\partial(\rho u_j)}{\partial t} + \sum_{i=1}^{3} \frac{\partial(\rho u_i u_j)}{\partial x_i} + \frac{\partial p}{\partial x_j} = 0$$

2. Potential Theory (Layzer) $$F_1 \frac{\ddot{\eta}}{D} + F_2 \frac{c^2 k^2 \dot{\eta}^2}{8D^2} + 2gA\eta_2 = 0$$

$$F_1(\eta_2), F_2(\eta_2), D(\eta_2)$$

$$\eta_2 = \eta_2(., \eta_0 k, e^{-k\eta})!$$

3. Potential Theory W/ $\eta_0 = \eta^*$

Magic: η_2 becomes Constant!

4. Potential Theory W/ $\eta_0 = \eta^*$ <u>and</u> Large g(t)

LEVEL 3

For $\eta \le \eta^*$**, use linear theory**

$$\ddot{\eta} - gkA\eta = 0$$ ***(Rayleigh, Taylor)***

For $\eta \ge \eta^*$**, use nonlinear theory**

$$F_1 \frac{\ddot{\eta}}{D} + F_2 \frac{c^2 k^2 \dot{\eta}^2}{8D^2} + 2gA\eta_2 = 0 \quad \Rightarrow \quad \ddot{\theta}_L - gk_L A_L \theta_L = 0$$

$$\theta_L \equiv e^{(\eta-\eta_0)k_L}, \quad \eta = \eta_0 + \frac{1}{k_L}\ln(\theta_L), \quad g = g(t),$$

$$\eta^* \equiv \frac{1}{k(1+c)} = 1/2k \ \textbf{or} \ 1/3k$$

LEVEL 3 $\rightarrow$ LEVEL 4

Define $s \equiv \int \sqrt{g(t)}dt$ ***, substitute in***

$$\ddot{\theta}_L - g k_L A_L \theta_L = 0$$ ***Level 3***

And get

$$\frac{d^2\theta_L}{ds^2} - k_L A_L \theta_L + \frac{1}{2g^2}\frac{dg}{dt}\frac{d\theta_L}{dt} = 0$$

Drop the last term and solve it

$$\theta_L = \cosh(s\sqrt{k_L A_L})$$ ***Level 4***

For ~all g(t).

EXAMPLES

g~constant $\Rightarrow$ ***RT, Level 4=Level 3, sol. known***

g~δ(t) $\Rightarrow$ ***RM, Level 4 sol. does not work (g=0)***
Level 3 sol. known.

g~t

Level 4≅Level 3, sols. known:

Level 4 sol.: $\cosh(s\sqrt{k_L A_L})$***, always***

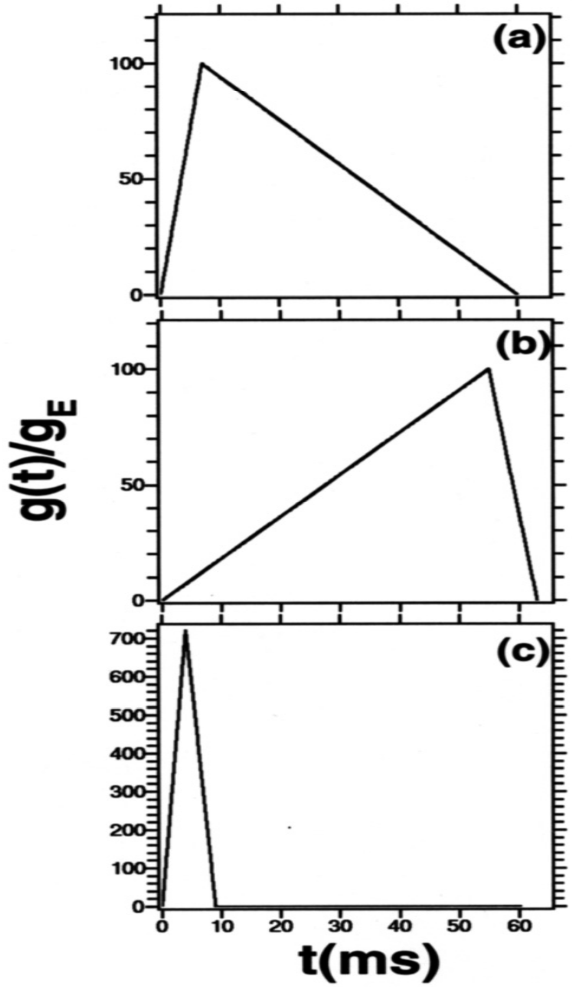
Level 3 sol.: Airy functions

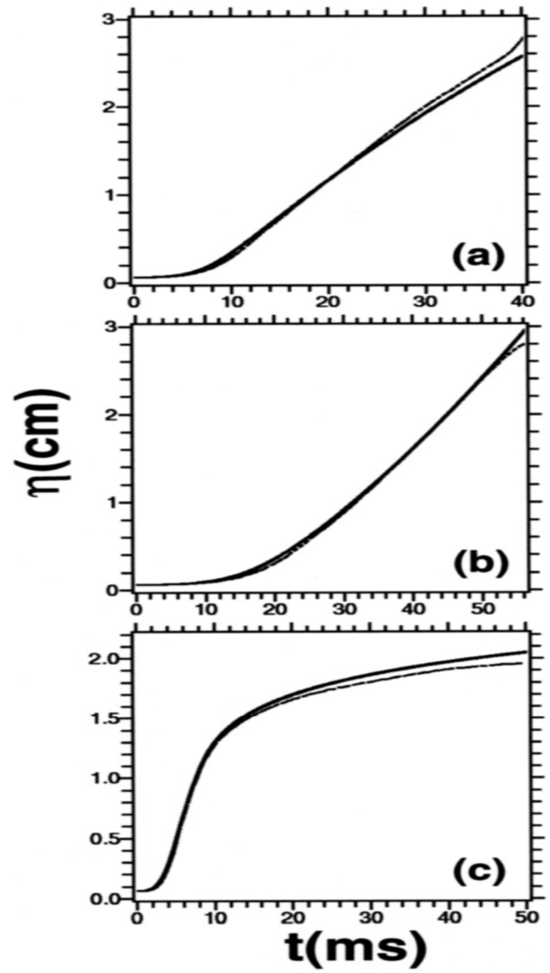
Other examples exist (PRE <u>81</u>, 016325 (2010)).

Similarity to Schrödinger's Eq.

CALE (Level 1) vs. ANALYTICS

LEM "exps." w/ 3 sinusoidal perturbations*



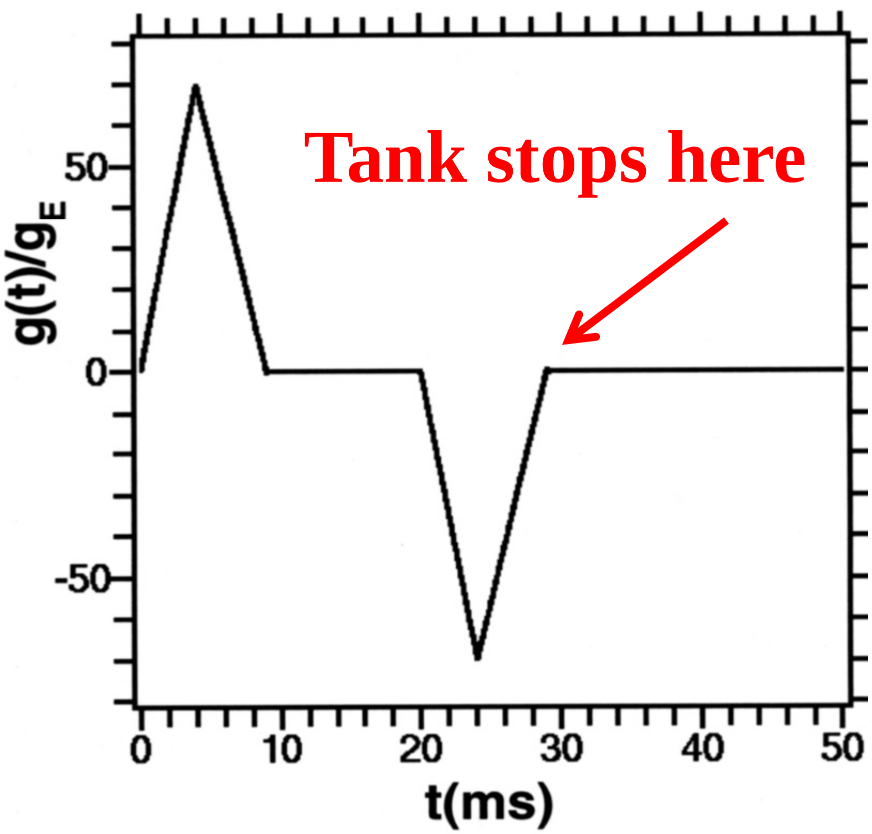


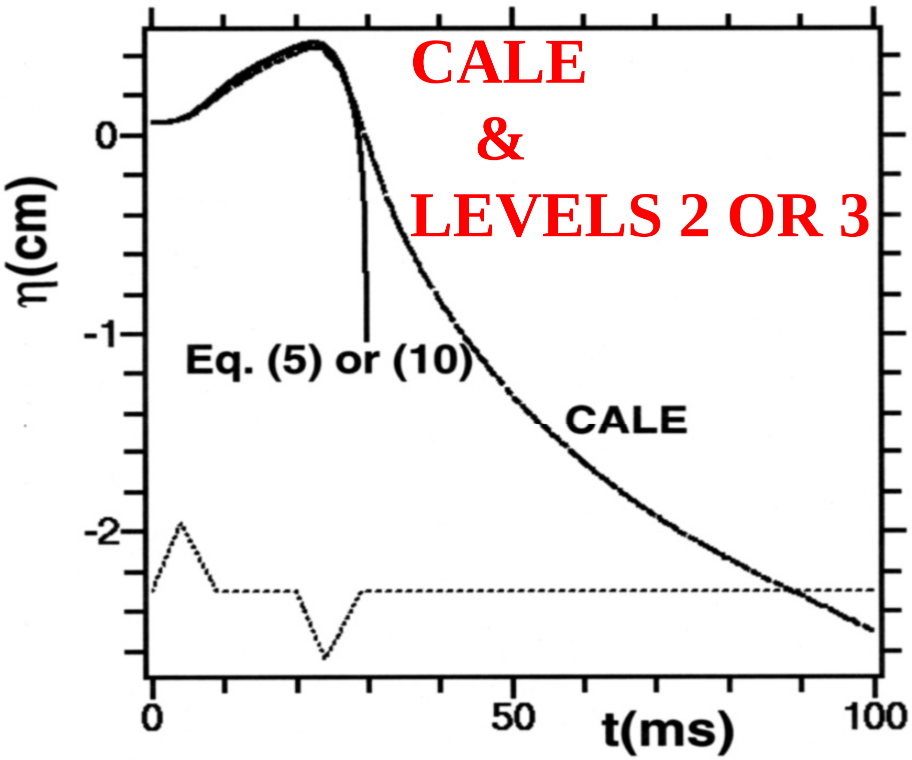
CALE
&
LEVEL 3

*Dimonte & Schneider PRE 54, 3740 (1996)

MODEL DOES NOT WORK WHEN g(t)<0

Two equal and opposite impulses ("shocks")

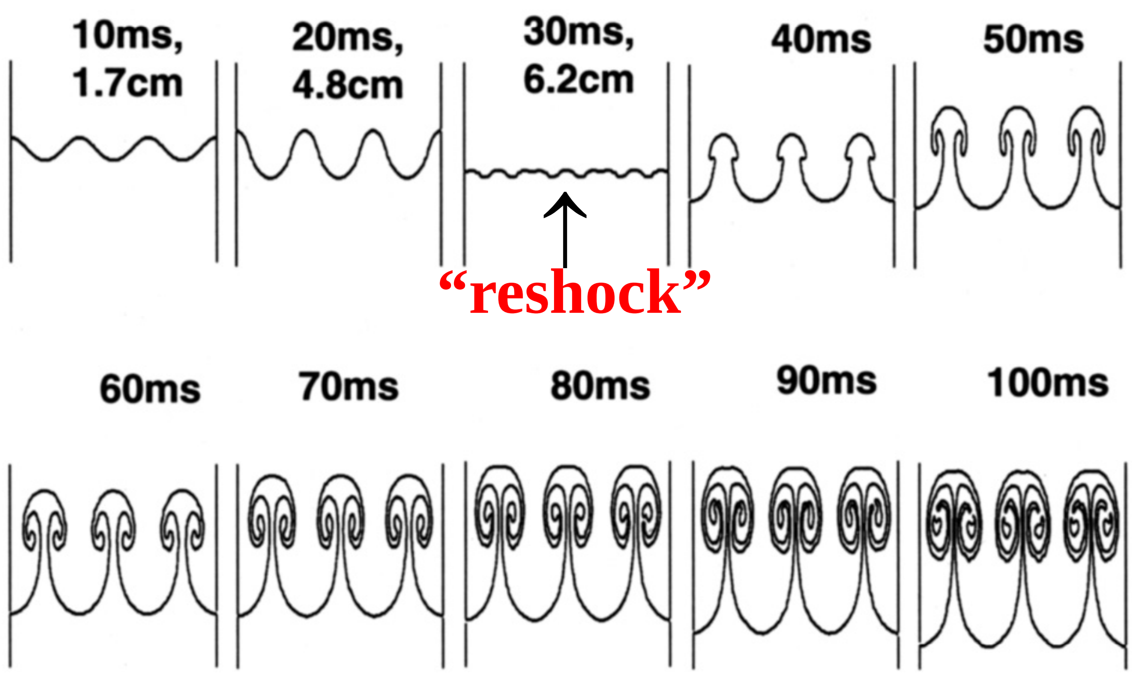




The MODEL WORKS ONLY FOR BUBBLES

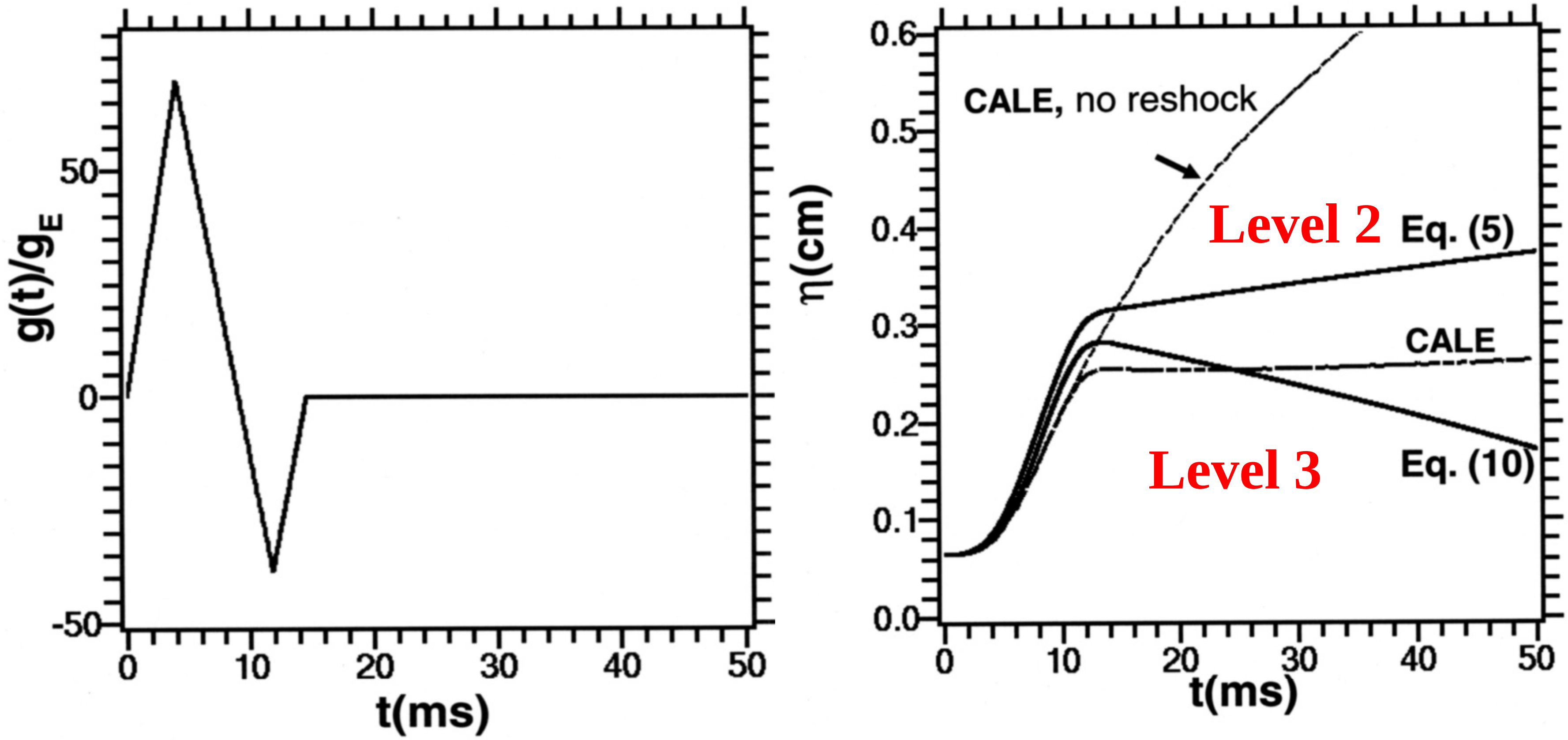
Two equal and opposite impulses

Bubbles ⇔ Spikes upon "reshock"



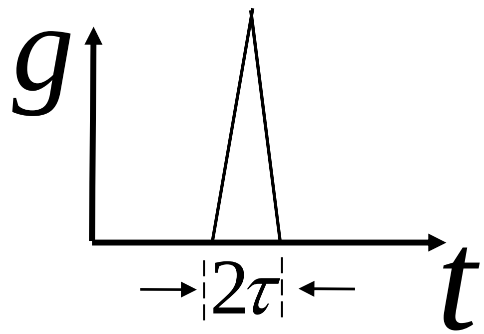
CALE

SMALL DIFFERENCE BETWEEN LEVELS 2 & 3



FREEZE-OUT WITH A WEAKER RESHOCK

IMPULSE ~ SHOCK



A narrow impulse acts like a shock.

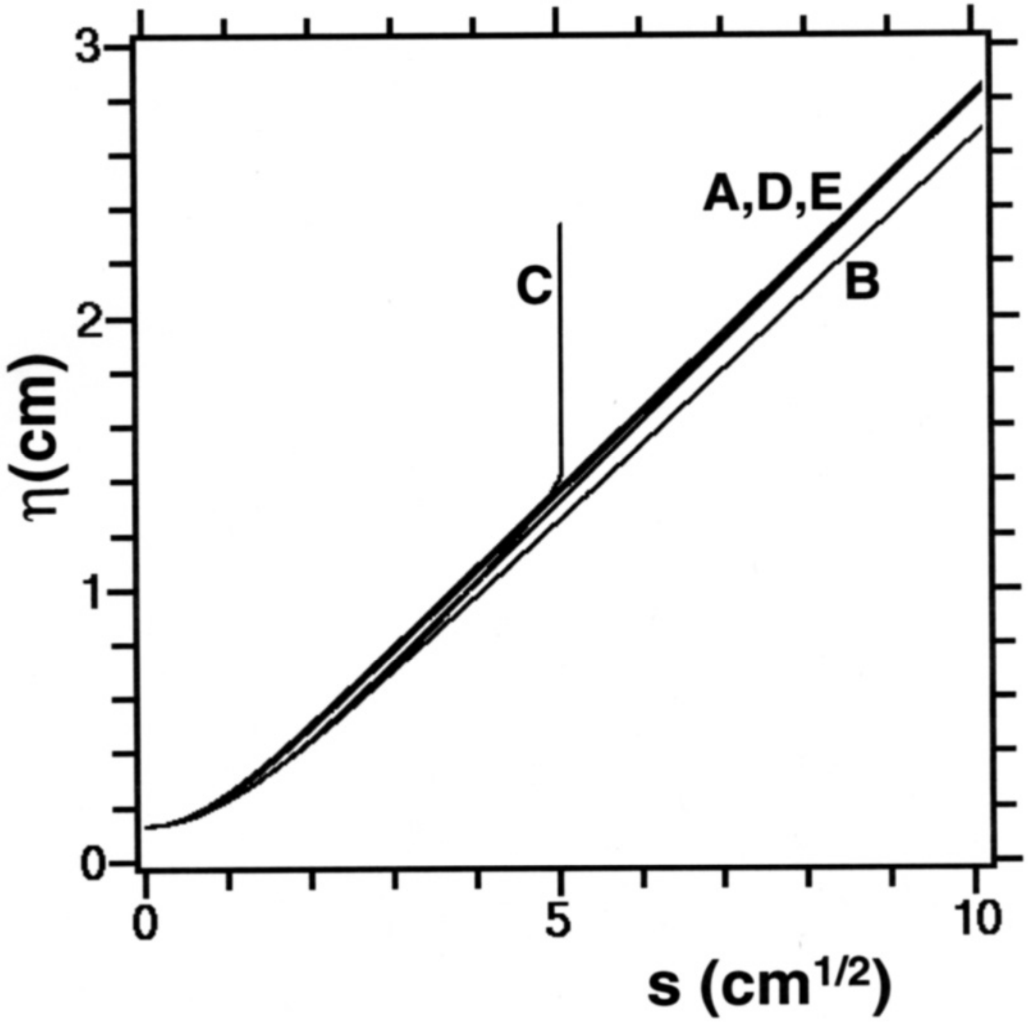
Corrections to Richtmyer's formula $(\dot{\eta} = \eta_0 \Delta \text{v} k A)$ due to the finite width (2τ) of the pulse:

$$\dot{\eta} = \eta_0 \Delta \text{v} k A (1 + 7 \Delta \text{v} k A \tau / 30 + \ldots)$$

This is an expansion. Exact solution also known.

TEST OF LEVEL 4 : SCALING WITH S

4 Different Accelerations w/ CALE (A, B, C, D)



One Level-4 curve (E)

$$\eta = \eta_0 + \frac{1}{k_L}\ln[\cosh(s\sqrt{k_L A_L})]$$

CONCLUSIONS and OTHER RESULTS

1. Levels 2-4 compare very well with Level 1 provided $g>0$ and $\eta=\eta_{bubble}$.
2. η_{bubble} scales w/ $s(t) \equiv \int \sqrt{g(t)}dt$.

   — — —

3. η_{spike} appears to scale w/ $\Delta x(t) \equiv \iint g(t')dt'dt$. Can show this for A=1 from Layzer's Eq.
4. *Zitterbewegung* (i.e., rapid oscillation of g(t) around an average value) has little effect on η.
5. Spikes for A<1 still need work (old problem).
6. Many interesting experiments (freeze-out, etc.) could have been performed w/ LEM.