A Vortex Model of Turbulence

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There are enough facts, both in theoretical calculations and experiments, which indicate that turbulent mixing is a vortex-type flow of discrete structures – rings, strings and fragments of strings. These facts justify a natural approach to simulating the turbulence process, namely, the vortex approach.

The equation for $\vec{\omega} = rot\vec{V}$,

$$\left(\frac{\partial}{\partial t} + (\vec{V} \cdot \nabla)\right)\frac{\vec{\omega}}{\rho} = \left(\frac{\vec{\omega}}{\rho} \cdot \nabla\right)\vec{V} + \frac{1}{\rho} \cdot rot\vec{A}, \tag{1}$$

where $A_i = div_k \Pi_{ik}$ is acceleration, follows from Navier-Stokes equations and the continuity equation. This is the equation for an instant value of **V**. Averaging with respect to the vortex tube's rotation period is performed, rather than averaging in fluctuations, as in stochastic models. The result is the kinetic equation for a case of longitudinal vortexes $\left(\vec{\omega} \,||\, \vec{V}\right)$ in approximation to small angles between $\vec{V}$ and $\vec{\omega}$ (2) and transversal vortexes with $\vec{\omega} \perp \vec{V}$ (3).

$$\frac{\partial \frac{f}{\rho}}{\partial t} + V \cdot \left(1 - \frac{\Theta^2}{2}\right) \cdot \frac{\partial \frac{f}{\rho}}{\partial z} = \frac{f}{\rho} \cdot div_V A + \frac{\alpha_0}{4} \cdot \frac{\partial \ln V}{\partial z} \cdot \frac{1}{\Theta} \cdot \frac{\partial}{\partial \Theta} \Theta \frac{\partial}{\partial \Theta} \frac{f}{\rho} + Q, \tag{2}$$

$$\left(\frac{\partial}{\partial t} + (\vec{V} \cdot \nabla)\right)\frac{\vec{\omega}}{\rho} = \frac{1}{\rho} \cdot rot\vec{A} \tag{3}$$

Θ is angle between $\vec{V}$ and $\vec{\omega}$, $f = f\left(\Theta, z\right)$ is density of vorticity.

Equation (3) has been implemented in a 2D hydrodynamic code. $Q = rot\left(-\frac{1}{\rho}\nabla p\right)$ is a source.

It has been found that drift velocity $V_{дрейфа} = -\vec{V} \times \frac{rot\vec{A}}{\omega^2}$ occurs with $\omega \neq const$. Equations (1), (2) are easy to use in analytical studies to make useful conclusions. The value of quantity $div_V \vec{A}$ determines, whether the flow of interest is stable ($div_V \vec{A} < 0$), or instable ($div_V \vec{A} > 0$). The case Q>0 (an absolutely instable shear flow) should be also taken into account. This criterion is valid for instabilities of all known kinds. There have been also found the cumulative instability and growth of a “hot spot” during the HE initiation process. Analytical and semi-analytical solutions have been constructed for a number of problems, such as

- the problem of a buoyant jet in an incompressible fluid. For this problem we have found the numerical value of α_0 in inverse path $\alpha_s = \alpha_0 \cdot \frac{\partial \ln V}{\partial z}$. The found value of α_0 has been used to solve some other problems;
- the problem of colliding jets;
- the problem of a shock wave ongoing a turbulent flow;
- the hot spot growth problem during HE initiation by a shock wave.

The results obtained are in a good agreement with data of experiments.

Since the kinetic equation is solved, the model provides a good description of the generation and evolution of turbulence on discontinuities of flow – shock waves, interfaces between materials of different densities and with shear.

2D approximation calculations have been carried out for mixing processes of various types.

Currently, the vortex model is used to develop a hot-spot model for the plastic flow of a solid and the low-velocity impact on a solid HE.