On Specification of Initial Conditions in Turbulence Models

Bertrand Rollin[1], Malcolm J. Andrews[2]

Los Alamos National Laboratory

[1] bertrand@lanl.gov, [2] mandrews@lanl.gov

LDRD: Turbulence by Design

LA-UR 10-04417

IWPCTM 12 - Moscow – 07/16/10

Turbulence "Control" via Initial Conditions

Hypothesis:

- Carefully prescribed initial conditions could be used to control "late-time" turbulent transport and mixing effectiveness.

Motivation:

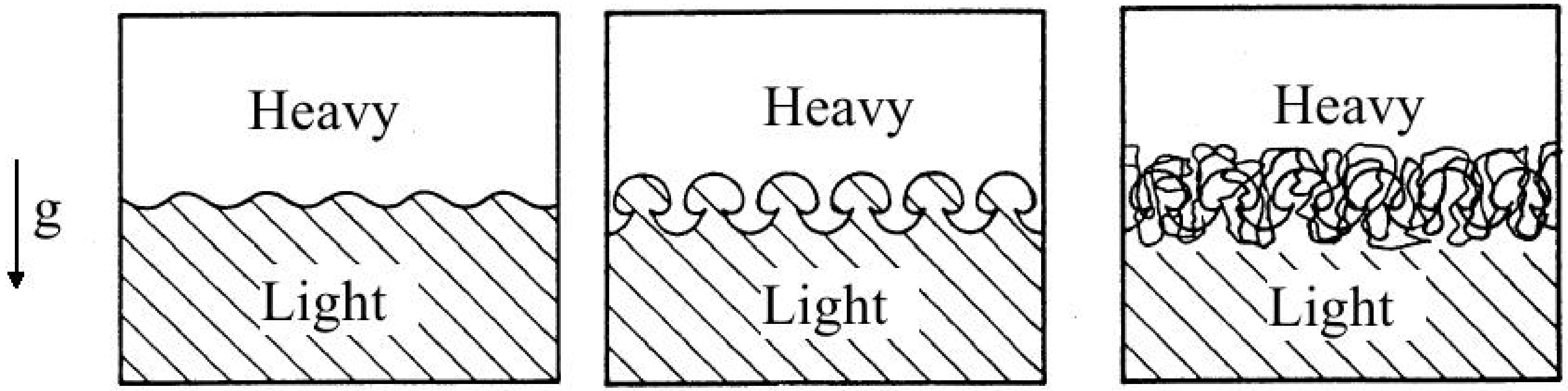
- Provide a rational basis for setting up initial conditions in turbulence models.

Objective:

- Predict profiles of relevant variables before the fully turbulent regime and use them as initial conditions for the turbulence model.

Rayleigh-Taylor Instability

Credit: M.J. Andrews



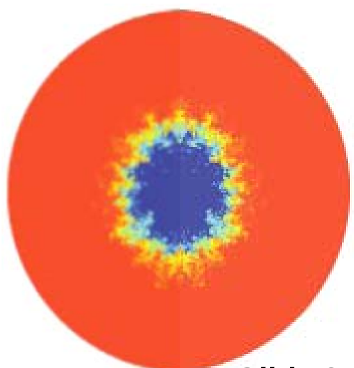
Exponential growth **Non-linear growth** **Turbulent mixing**

Characteristic non-dimensional number: $A_T = \dfrac{\rho^h - \rho^l}{\rho^h + \rho^l}$

Interface is unstable if: $\nabla p \cdot \nabla \rho < 0$

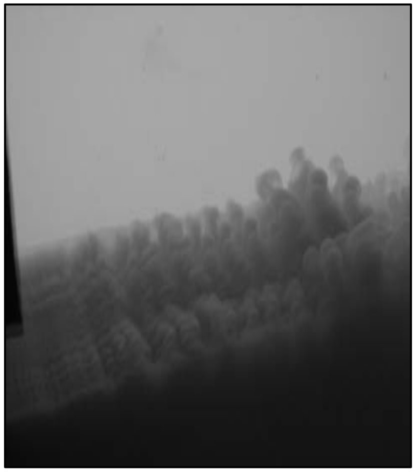
Baroclinic generation of vorticity: $\dfrac{1}{\rho^2} \nabla p \times \nabla \rho$

Inertial Confinement Fusion (ICF)



Some Dramatic Effects of Initial Conditions

M.J. Andrews, TAMU water channel experiment

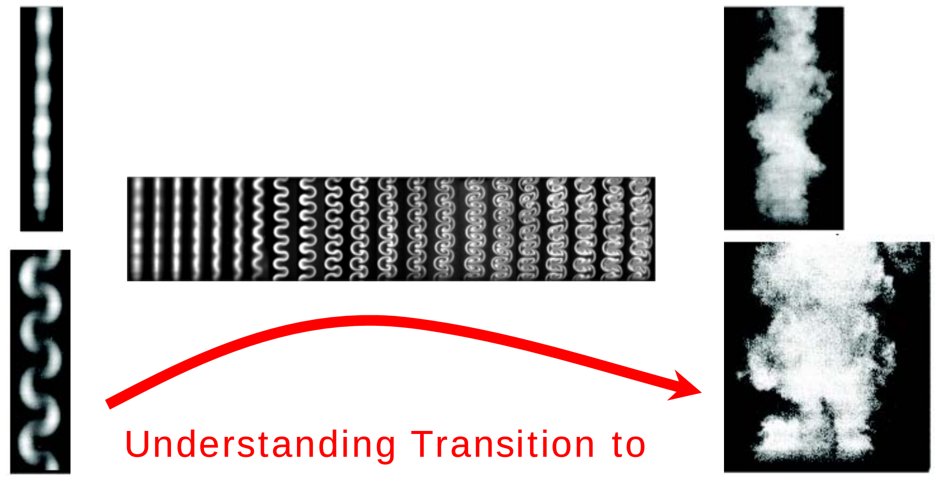


Long wavelength initial conditions

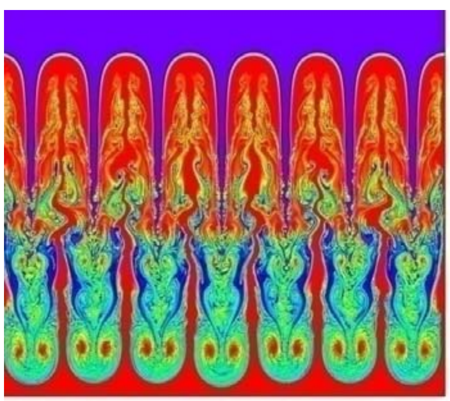
Short wavelength initial conditions

Richtmyer-Meshkov (RM) Transitions From Different Initial Conditions

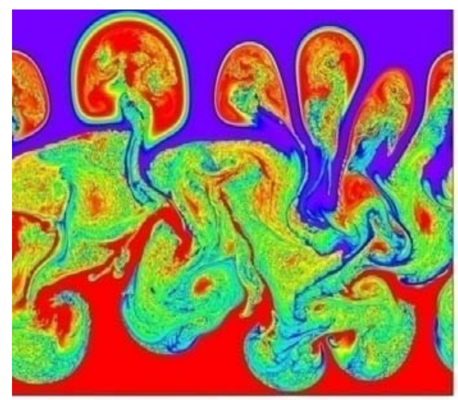
(from the LANL Gas Shock Tube – K. Prestridge)



Credit: Hjelm & Ristorcelli



No IC noise



With IC noise

BHR Turbulence Model for RT Instability

Besnard-Harlow-Rauenzhan (BHR) turbulence model:

- **Single-point turbulent transport model**
- **Designed for variable density turbulence**

D. Besnard, F. H. Harlow, R. Rauenzhan, LA-10911-MS (1987)

Model Variables:

$$k = \frac{1}{2}\overline{u_i' u_i'} \qquad a_i = \frac{\overline{\rho' u_i'}}{\bar{\rho}} \qquad b = -\overline{\rho' v'} \qquad S = \frac{k^{3/2}}{\varepsilon} \qquad \nu_t = C_\mu k^{1/2} S$$

Governing equation for the variable S:

$$\partial_t S = \left(\frac{3}{2} - C_4\right) a_z g \frac{S}{k} + \frac{1}{\rho}\partial_z\left(\rho \frac{\nu_t}{\sigma_S}\partial_z S\right) - \left(\frac{3}{2} - C_2\right) k^{1/2}$$

BHR initiated with:

- **Profiles for:** $k \quad a_i \quad b \quad S$
- **Values for:** $C_4 \quad C_2 \quad C_\mu \quad \sigma_S \quad \ldots$

Procedure for Determining ICs for BHR

Heights and velocities of bubbles and spikes fronts

↓

Fluids volume fraction profiles

↓

Profiles of BHR variables

An ODE Model for Multi-mode

Goncharov model:

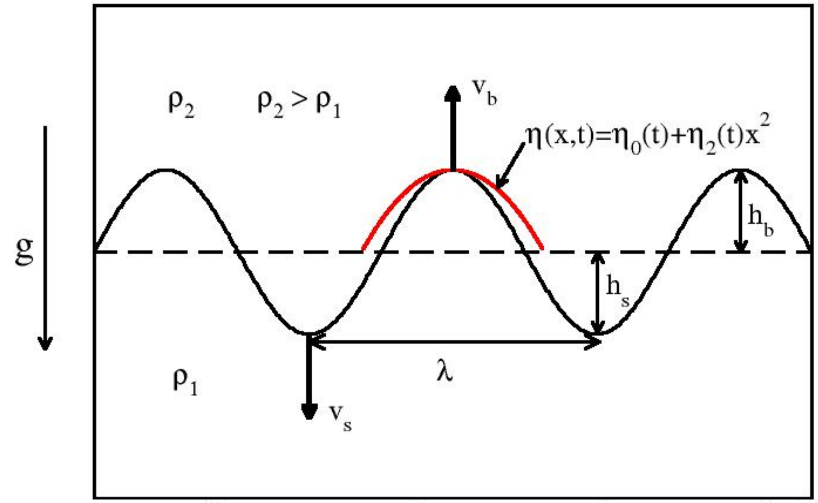
- **Velocity potentials (3D bubble)**

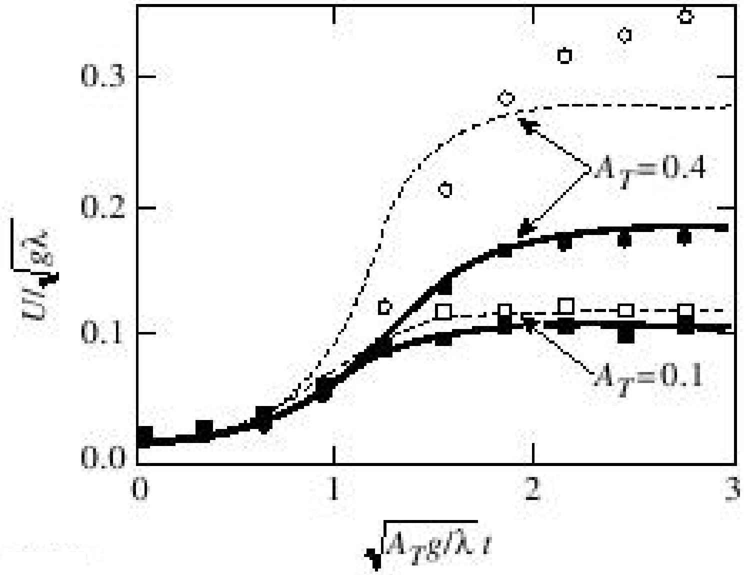
$$\phi^h = a(t)J_0(kr)e^{-k(z-\eta_0)}$$

$$\phi^l = b_1(t)J_0(kr)e^{k(z-\eta_0)} + b_2(t)z$$

- **Non-linear model**
- **Valid on a large range of A_T**
- **Good prediction for bubble**

- **Single mode model**
- **Spike inaccurate for high A_T**





Multi-mode model:

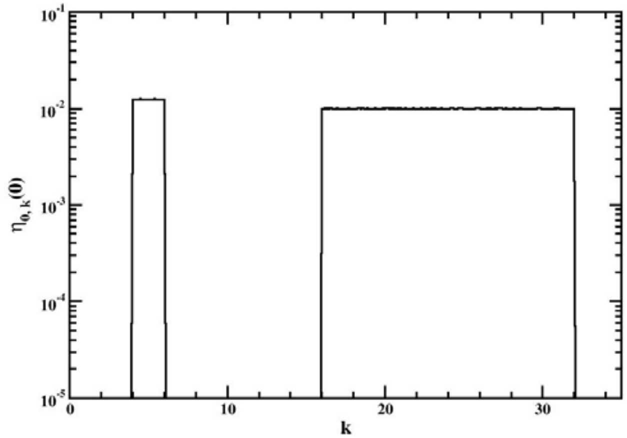
$$\boxed{h_b(t) = \max_k(h_{b,k}(t))}$$

Goncharov, PRL, **88**, 2002

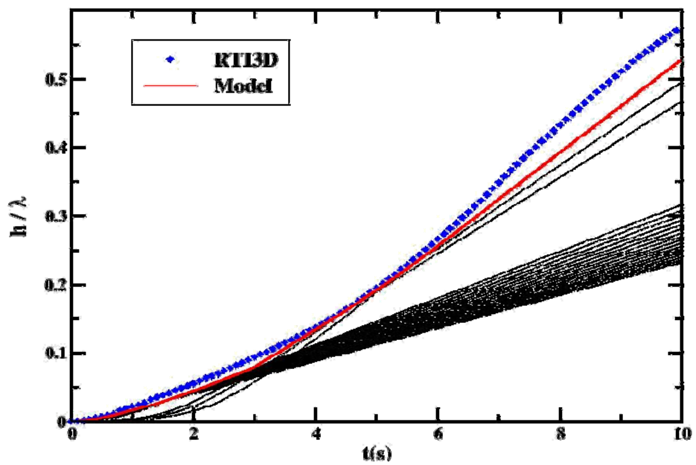
Multimode Model Performance

- **Simulation made with RTI3D**
- **Atwood:** $A_T = 0.01$
- **Gravity:** $g = 40$
- **Simulation size:** $64 \times 64 \times 160$

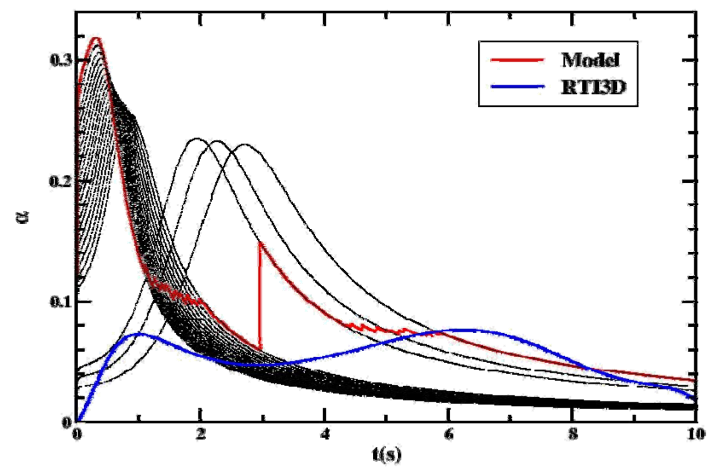
Initial Perturbation Spectrum



Bubble Front Evolution



Growth Rate Evolution



Our model captures key features of the bubble's front evolution

Procedure for Determining ICs for BHR

Heights and velocities of bubbles and spikes fronts

↓

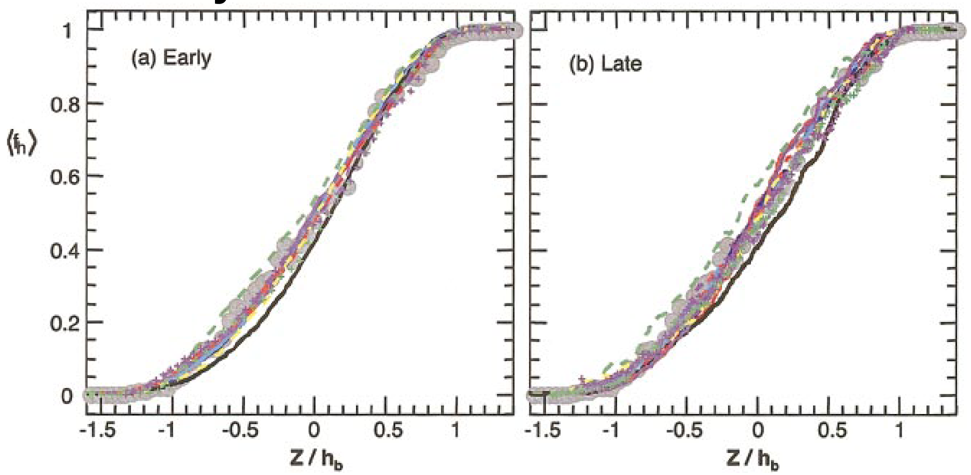
Fluids volume fraction profiles

↓

Profiles of BHR variables

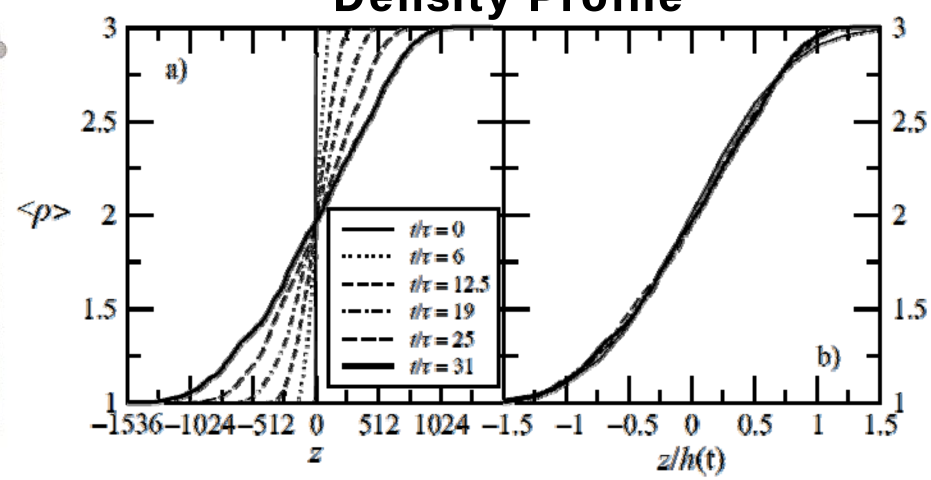
Linear Approximation for Density Profile

Heavy Fluid Volume Fraction Profile



Dimonte *et al*., Phys. of Fluids, **16** (2004)

Density Profile



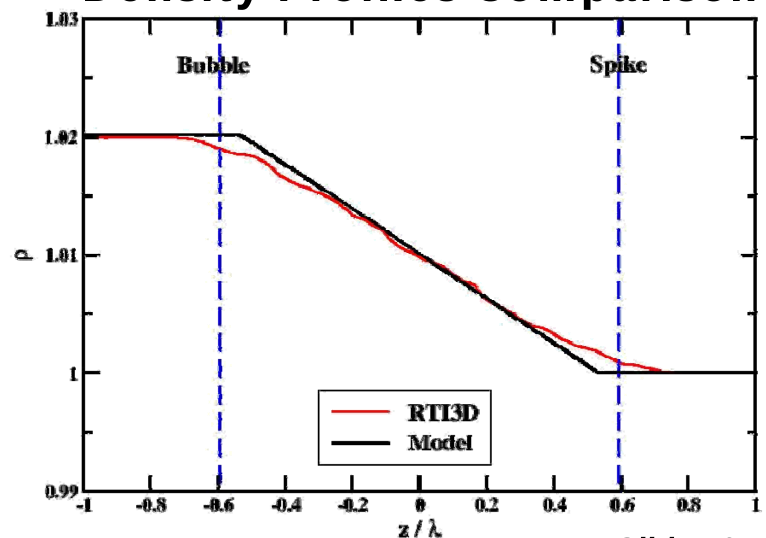
Livescu *et al*., J. Turbulence, **10** (2009)

$$\rho = f_l \rho_l + f_h \rho_h$$

$$\begin{cases} f_l = \dfrac{\rho - \rho_h}{\rho_l - \rho_h} \\ f_h = 1 - f_l \end{cases}$$

$$\boxed{\begin{cases} f_h = 0 & \text{if} & z < -h_s \\ f_h = 0.5\dfrac{z + h_s}{h_s} & \text{if} & -h_s \le z < 0 \\ f_h = 0.5\dfrac{z}{h_b} + 0.5 & \text{if} & 0 \le z \le h_b \\ f_h = 1 & \text{if} & z > h_b \end{cases}}$$

Density Profiles Comparison



Procedure for Determining ICs for BHR

Heights and velocities of bubbles and spikes fronts

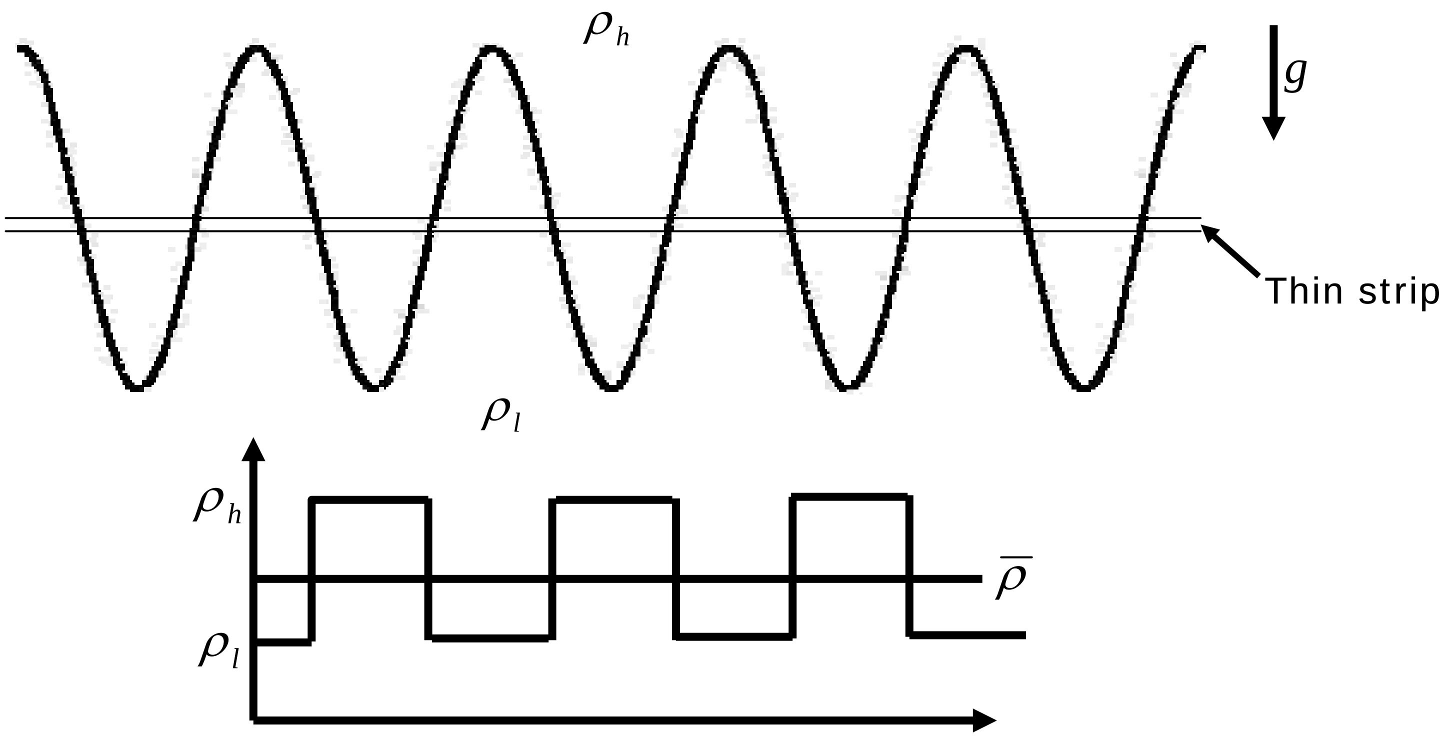
↓

Fluids volume fraction profiles

↓

Profiles of BHR variables

Two-Fluid Model



$$\bar{\rho} = \alpha_l \rho_l + \alpha_h \rho_h$$

$$\bar{\mathbf{u}} = \alpha_l \mathbf{u}_l + \alpha_h \mathbf{u}_h$$

Steinkamp, LA-13123-T Thesis (1996)

Two-Fluid Formulation for BHR Variables

$$k = C_k \frac{3}{2}\left(\vec{v}_b - \vec{v}_s\right)^2 \frac{f_h f_l \rho_h \rho_l}{\left(f_h \rho_h + f_l \rho_l\right)^2}$$

Isotropy hypothesis

$$a_z = C_{a_z} \frac{f_h f_l}{f_h \rho_h + f_l \rho_l} \left(\rho_h - \rho_l\right)\left(\vec{v}_s - \vec{v}_b\right)$$

$$b = C_b \frac{f_h f_l \left(\rho_h - \rho_l\right)^2}{\rho_h \rho_l}$$

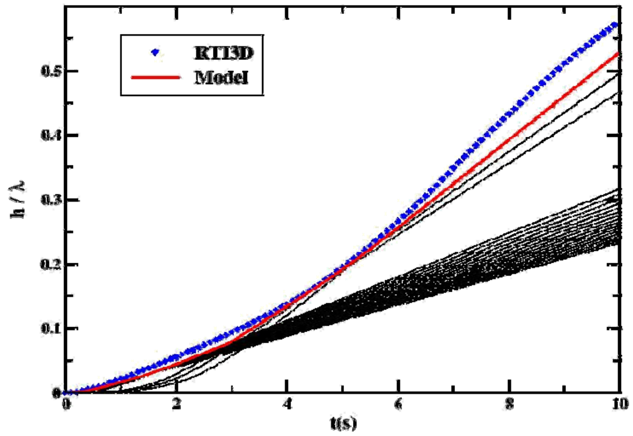
$$S = C_S \left(h_b + h_s\right)\left(4 f_h f_l\right)^{1/2}$$

- **Self-similarity hypothesis**
- **Derived for low Atwood number**

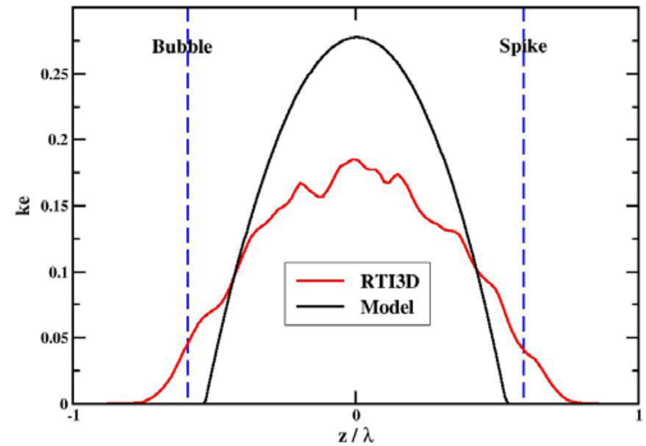
$$C_k = C_S = C_b = C_{a_z} = 1$$

Preliminary Results

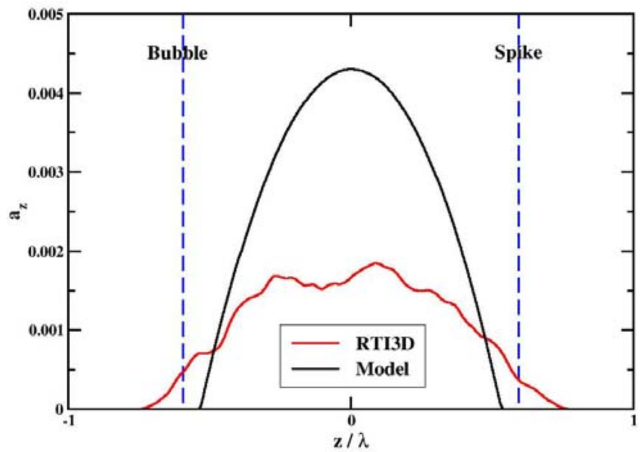
Bubble Front Evolution



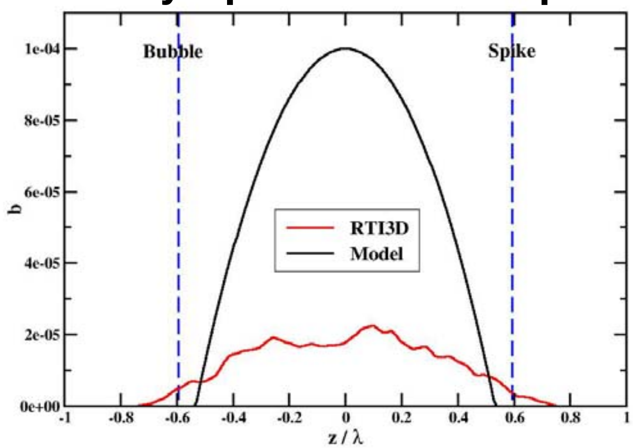
Kinetic Energy Profile



Normalized Mass Flux Profile



Density-specific volume profile



Two-fluid formulation gives interesting results without adjusting correction coefficients

Conclusions

- **Summary:**
  - A work in progress
  - An approach for setting up ICs in a turbulence model
  - An ODE that captures the development of initial power spectrum
  - Two-fluid formulation for BHR variables profiles

- **Future work:**
  - Improve/derive multi-mode model for bubbles/spikes front
  - Dynamic BHR coefficient based on self-similar solution
  - Adjust model coefficients
  - Extend model for all Atwood number

- **Acknowledgements:**

This work is funded by the Laboratory Directed Research and Development Program at Los Alamos National Laboratory through directed research project number LDRD-20090058DR.

Self-Similar Solution for "Dynamic" C_4 Derivation

$$k = \alpha_k A_T^2 g^2 t^2 \qquad a_z = \alpha_{a_z} A_T g t \qquad S = \alpha_S A_T g t^2$$

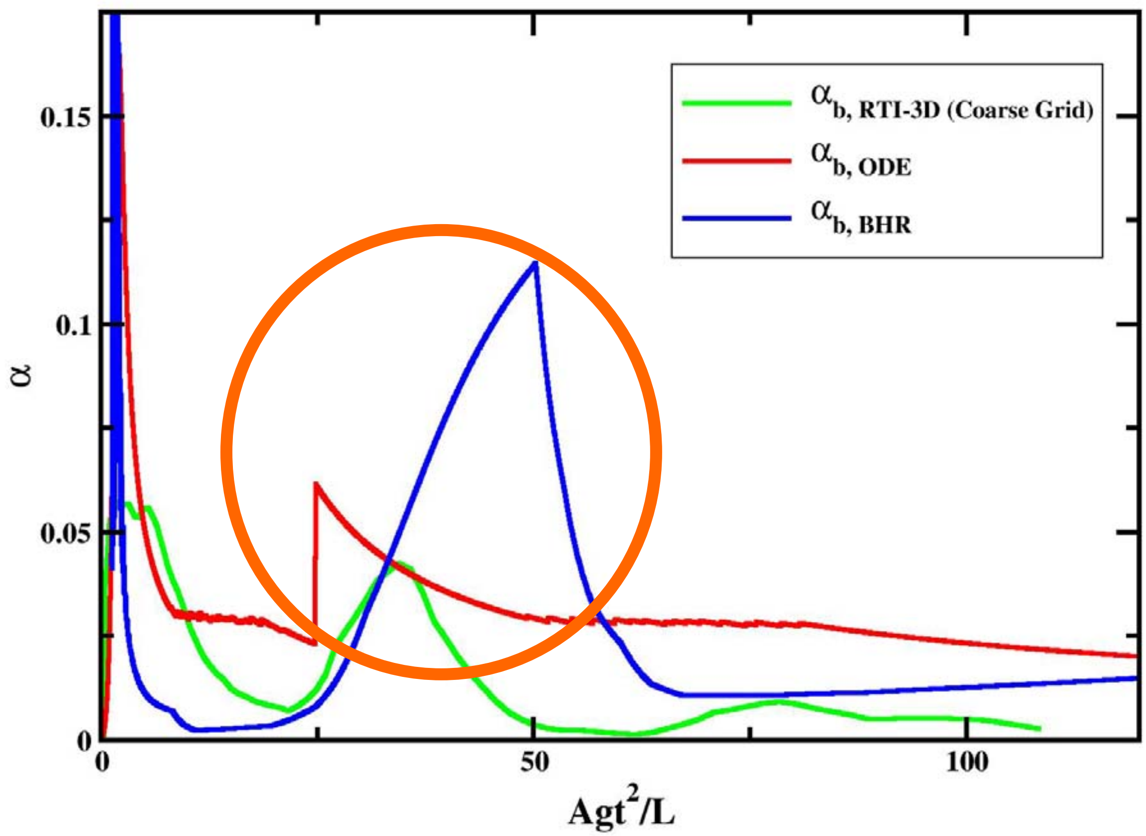
$$\Rightarrow \quad \alpha_k = \frac{\frac{d^2 k}{dt^2}}{2A_T^2 g^2}, \alpha_a = \frac{\frac{da}{dt}}{A_T g}, \alpha_s = \frac{\frac{d^2 S}{dt^2}}{2A_T g}$$

$$\partial_t S = \left(\frac{3}{2} - C_4\right) a_z g \frac{S}{k} + \frac{1}{\rho}\partial_z\left(\rho \frac{\nu_t}{\sigma_S}\partial_z S\right) - \left(\frac{3}{2} - C_2\right) k^{1/2}$$

$$2\alpha_S = \left(\frac{3}{2} - C_4\right)\frac{\alpha_{az}\alpha_S}{\alpha_k A} - \left(\frac{3}{2} - C_2\right)\alpha_k^{1/2}$$

$$\Rightarrow \quad \boxed{C_4 = f(\alpha_k, \alpha_S, \alpha_{a_z}, C_2)}$$

Preliminary Result with “Dynamic” C4



- **BHR captures dynamics of α for a banded spectrum with “dynamically” prescribed C_4**

Governing Equations for 1D BHR Model

$$\partial_t k = a_z g + \frac{1}{\rho}\partial_z\left(\rho\frac{\nu_t}{\sigma_k}\partial_z k\right) - \frac{k^{3/2}}{S}$$

$$\partial_t a_z = bg - \frac{2}{3}\frac{k}{\rho}\partial_z\rho + \frac{1}{\rho}\partial_z\left(\rho\frac{\nu_t}{\sigma_{a_z}}\partial_z a_z\right) - C_{a1}\frac{k^{1/2}}{S}a_z$$

$$\partial_t b = -2(b+1)\frac{a_z}{\rho}\partial_z\rho + \frac{1}{\rho}\partial_z\left(\rho\frac{\nu_t}{\sigma_b}\partial_z b\right) - C_{b2}\frac{k^{1/2}}{S}b$$

$$\partial_t S = \left(\frac{3}{2} - C_4\right)a_z g\frac{S}{k} + \frac{1}{\rho}\partial_z\left(\rho\frac{\nu_t}{\sigma_S}\partial_z S\right) - \left(\frac{3}{2} - C_2\right)k^{1/2}$$

$$\partial_t\rho = \partial_z\left(\rho\frac{\nu_t}{\sigma_t}\partial_z\rho\right) \qquad \nu_t = C_\mu k^{1/2}S$$