On Specification of Initial Conditions in Turbulence Models

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LDRD: Turbulence by Design

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Turbulence “Control” via Initial Conditions

Hypothesis:

- Carefully prescribed initial conditions could be used to control “late-time” turbulent transport and mixing effectiveness.

Motivation:

- Provide a rational basis for setting up initial conditions in turbulence models.

Objective:

- Predict profiles of relevant variables before the fully turbulent regime and use them as initial conditions for the turbulence model.
Rayleigh-Taylor Instability

Characteristic non-dimensional number: 

\[ A_T = \frac{\rho^h - \rho^l}{\rho^h + \rho^l} \]

Interface is unstable if: \( \nabla p \cdot \nabla \rho < 0 \)

Baroclinic generation of vorticity: 

\[ \frac{1}{\rho^2} \nabla p \times \nabla \rho \]

Inertial Confinement Fusion (ICF)
Some Dramatic Effects of Initial Conditions

M.J. Andrews, TAMU water channel experiment

Richtmyer-Meshkov (RM) Transitions From Different Initial Conditions
(from the LANL Gas Shock Tube – K. Prestridge)

Long wavelength initial conditions
Short wavelength initial conditions

Understanding Transition to Turbulence

Credit: Hjelm & Ristorcelli

No IC noise
With IC noise
BHR Turbulence Model for RT Instability

Besnard-Harlow-Rauenzhan (BHR) turbulence model:
- Single-point turbulent transport model
- Designed for variable density turbulence


Model Variables:
\[
k = \frac{1}{2} u_i' u_i' \quad a_i = \frac{\rho' u_i'}{\bar{\rho}} \quad b = -\rho' v' \quad S = \frac{k^{3/2}}{\varepsilon} \quad \nu_t = C_\mu k^{1/2} S
\]

Governing equation for the variable S:
\[
\partial_t S = \left(\frac{3}{2} - C_4 \right) a_z g \frac{S}{k} + \frac{1}{\rho} \partial_z \left( \rho \frac{\nu_t}{\sigma_S} \partial_z S \right) - \left(\frac{3}{2} - C_2 \right) k^{1/2}
\]

BHR initiated with:
- Profiles for: \( k \quad a_i \quad b \quad S \)
- Values for: \( C_4 \quad C_2 \quad C_\mu \quad \sigma_S \quad ... \)
Procedure for Determining ICs for BHR

1. Heights and velocities of bubbles and spikes fronts
2. Fluids volume fraction profiles
3. Profiles of BHR variables
An ODE Model for Multi-mode

Goncharov model:

- Velocity potentials (3D bubble)
  \[ \phi^h = a(t)J_0(kr)e^{-k(z-\eta_0)} \]
  \[ \phi^l = b_1(t)J_0(kr)e^{k(z-\eta_0)} + b_2(t)z \]

- Non-linear model
- Valid on a large range of $A_T$
- Good prediction for bubble
- Single mode model
- Spike inaccurate for high $A_T$

Multi-mode model:

\[ h_b(t) = \max_k(h_{b,k}(t)) \]

Goncharov, PRL, 88, 2002

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Multimode Model Performance

- Simulation made with RTI3D
- Atwood: $A_T = 0.01$
- Gravity: $g = 40$
- Simulation size: $64 \times 64 \times 160$

Our model captures key features of the bubble’s front evolution
Procedure for Determining ICs for BHR

1. Heights and velocities of bubbles and spikes fronts
2. Fluids volume fraction profiles
3. Profiles of BHR variables
Linear Approximation for Density Profile

\[ \rho = f_l \rho_l + f_h \rho_h \]

\[
\begin{align*}
  f_l &= \frac{\rho_l - \rho_h}{\rho_l - \rho_h} \\
  f_h &= 1 - f_l \\
  f_h &= 0 & \text{if} & z < -h_s \\
  f_h &= 0.5 \frac{z + h_s}{h_s} & \text{if} & -h_s \leq z < 0 \\
  f_h &= 0.5 \frac{z}{h_b} + 0.5 & \text{if} & 0 \leq z \leq h_b \\
  f_h &= 1 & \text{if} & z > h_b
\end{align*}
\]


Livescu et al., J. Turbulence, 10 (2009)
Procedure for Determining ICs for BHR

- Heights and velocities of bubbles and spikes fronts
- Fluids volume fraction profiles
- Profiles of BHR variables
Two-Fluid Model

\[ \overline{\rho} = \alpha_l \rho_l + \alpha_h \rho_h \]

\[ \overline{\mathbf{u}} = \alpha_l \mathbf{u}_l + \alpha_h \mathbf{u}_h \]

Two-Fluid Formulation for BHR Variables

\[ k = C_k \frac{3}{2} \left( \vec{v}_b - \vec{v}_s \right)^2 \frac{f_h f_l \rho_h \rho_l}{(f_h \rho_h + f_l \rho_l)^2} \]

\[ a_z = C_{a_z} \frac{f_h f_l}{f_h \rho_h + f_l \rho_l} (\rho_h - \rho_l) (\vec{v}_s - \vec{v}_b) \]

\[ b = C_b \frac{f_h f_l (\rho_h - \rho_l)^2}{\rho_h \rho_l} \]

\[ S = C_S \left( h_b + h_s \right) (4 f_h f_l)^{1/2} \]

- Isotropy hypothesis
- Self-similarity hypothesis
- Derived for low Atwood number

\[ C_k = C_S = C_b = C_{a_z} = 1 \]
Two-fluid formulation gives interesting results without adjusting correction coefficients
Conclusions

Summary:
- A work in progress
- An approach for setting up ICs in a turbulence model
- An ODE that captures the development of initial power spectrum
- Two-fluid formulation for BHR variables profiles

Future work:
- Improve/derive multi-mode model for bubbles/spikes front
- Dynamic BHR coefficient based on self-similar solution
- Adjust model coefficients
- Extend model for all Atwood number

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Self-Similar Solution for “Dynamic” C_4 Derivation

\[ k = \alpha_k A_T^2 g^2 t^2 \quad a_z = \alpha_{a_z} A_T g t \quad S = \alpha_S A_T g t^2 \]

\[ \alpha_k = \frac{d^2 k}{d t^2}, \quad \alpha_a = \frac{d a}{d t}, \quad \alpha_s = \frac{d^2 S}{d t^2} \]

\[ \partial_t S = \left( \frac{3}{2} - C_4 \right) a_z g \frac{S}{k} + \frac{1}{\rho} \partial_z \left( \rho \frac{v_t}{\sigma_S} \partial_z S \right) - \left( \frac{3}{2} - C_2 \right) k^{1/2} \]

\[ 2\alpha_s = \left( \frac{3}{2} - C_4 \right) \frac{\alpha_{a_z} \alpha_s}{\alpha_k A} - \left( \frac{3}{2} - C_2 \right) \alpha_k^{1/2} \]

\[ C_4 = f(\alpha_k, \alpha_s, \alpha_{a_z}, C_2) \]
BHR captures dynamics of $\alpha$ for a banded spectrum with “dynamically” prescribed $C_4$
Governing Equations for 1D BHR Model

\[
\begin{align*}
\partial_t k &= a_z g + \frac{1}{\rho} \partial_z \left( \rho \frac{v_t}{\sigma_k} \partial_z k \right) - \frac{k^{3/2}}{S} \\
\partial_t a_z &= b g - \frac{2}{3} \frac{k}{\rho} \partial_z \rho + \frac{1}{\rho} \partial_z \left( \rho \frac{v_t}{\sigma_{a_z}} \partial_z a_z \right) - C_{a1} \frac{k^{1/2}}{S} a_z \\
\partial_t b &= -2(b+1) \frac{a_z}{\rho} \partial_z \rho + \frac{1}{\rho} \partial_z \left( \rho \frac{v_t}{\sigma_b} \partial_z b \right) - C_{b2} \frac{k^{1/2}}{S} b \\
\partial_t S &= \left( \frac{3}{2} - C_4 \right) a_z g \frac{S}{k} + \frac{1}{\rho} \partial_z \left( \rho \frac{v_t}{\sigma_S} \partial_z S \right) - \left( \frac{3}{2} - C_2 \right) k^{1/2} \\
\partial_t \rho &= \partial_z \left( \rho \frac{v_t}{\sigma_t} \partial_z \rho \right) \\
v_t &= C_\mu k^{1/2} S
\end{align*}
\]