A subgrid scale model accounting for rapid distortion and spectral equilibrium limits in variable density flows : application to shock tube experiments

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Since the pioneering work of Youngs [1], large eddy simulation (LES) has gained interest as a tool for predicting the turbulent variable density flows encountered in shock tube experiments. In these experiments, the turbulent field is submitted to rapid compression and expansions, as well as large accelerations creating stable or unstable stratifications. These interactions can be strong enough to affect the small scales of turbulence and may lead to a state of rapidly distorted turbulence (RDT). On the other hand, in between interactions, turbulence is left to decay and diffuse, so that it tends to a state of spectral equilibrium.

As stressed in several works [2, 3, 4], the small scales of turbulence – and consequently the subgrid scale (SGS) model accounting for their presence in LES – behave differently whether in the RDT or in the spectral equilibrium regime.

In the spectral equilibrium limit, the small scale part of the turbulent spectrum can be shown to depend linearly on mean gradients [5]. By integrating this equilibrium spectrum, Smagorinsky-like SGS models are then obtained and justified [6]. Such derivations have essentially been applied to the context of constant density flows. An extension to stratified flows was given in [7], but with an erroneous equilibrium spectrum [5]. Besides, it seems that no derivation takes into account the effects of a mean compression/expansion.

As for the RDT limit, several studies [2, 3, 4] point out that Smagorinsky-like models fail to reproduce the main behavior of small turbulent scales. Scale-similarity and mixed models [8], on the other hand, are shown to perform better. Most of these conclusions are based on observations from experiments and direct simulations. However, they are not sustained by any theory. To the authors' knowledge, there is no derivation of a SGS model explicitly based on RDT assumptions.

Thus, the aim of this work is to derive a SGS model coherent with the RDT limit, and also with the spectral equilibrium limit when mean stratification and compression/expansion are applied.

For this purpose, two idealized problems are considered, each corresponding to one of the two above mentioned asymptotic limits. In the first case, an initially homogeneous and isotropic turbulence with density fluctuations is subjected to a rapid distortion. This distortion may be caused by strain, compression or stratification. A formal solution for the SGS fluxes and variances of velocity and density is established under RDT assumptions. Then, this exact solution is shown to be matched by a scale-similarity model. This scale-similarity model is different from the usual one derived in [8], as it is based on the Leonard tensor of the filtered velocity and density gradients.

The second case also consists in an initially homogeneous and isotropic turbulence with density fluctuations, but this time subjected to an infinitely weak distortion. An asymptotic analysis of the spectral model of Canuto & Dubovikov [9] is performed in this limit. A SGS model is deduced by integrating the resulting spectra. Smagorinsky-like expressions are obtained.

Finally, the general case is treated by making a heuristic assumption. An arbitrary limit Froude number $\mathcal{F}_l$ is introduced. Scales having a Froude number smaller than $\mathcal{F}_l$ are assumed to be in the RDT regime. Otherwise, they are assumed to be in equilibrium. Each part of the spectrum is modeled accordingly, making use of the two SGS models derived previously. As a result, a mixed model is obtained, that naturally accounts for RDT and spectral equilibrium limits.

In order to validate the derived SGS model, *a priori* tests are performed with the DNS code Triclade. The first test consists in the interaction of an expansion wave with a homogeneous and isotropic mixture initially at rest. By varying the intensity of the wave, RDT conditions as well as spectral equilibrium conditions can be met. This allows to verify the two building blocks of the model independently. A second test consisting in a Rayleigh-Taylor configuration is also performed.

References

[1] D. Youngs. *Laser Part. Beams*, 12:725-750, 1994.
[2] L. Shao, S. Sarkar and C. Pantano. *Phys. Fluids*, 11:1229-1248, 1999.
[3] S. Liu, J. Katz and C. Meneveau. *J. Fluid Mech.*, 387:281-320, 1999.
[4] J, Chen, J. Katz and C. Meneveau. *J. Fluid Eng.*, 127:840-849, 2005.
[5] T. Ishihara, K. Yoshida, and Y. Kaneda. *Phys. Rev. Lett.*, 88:154501, 2002.
[6] Y. Li and C. Meneveau. *Phys. Fluids*, 16:3483-3486, 2004.
[7] A. Yoshizawa. *J. Phys. Soc. Jap.*, 52:1194-1205, 1983.
[8] J. Bardina, J.H. Ferziger, W.C. Reynolds *AIAA paper*, 80:1357, 1980.
[9] V.M. Canuto and M.S. Dubovikov *Phys. Fluids*, 8:571-586, 1995.