A subgrid scale model accounting for rapid distortion and spectral equilibrium limits in variable density flows

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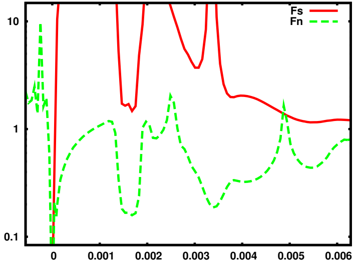
Context :

Large Eddy Simulation of shock tube experiments

- In shock tubes, several turbulent regimes can be observed :



DIF shock tube experiment (Haas, 2000) : Froude number at the mixing zone center

$$\mathcal{F}_s = \frac{1}{\mathcal{S}\tau} \quad , \quad \mathcal{F}_n = \frac{1}{\mathcal{N}\tau}$$

τ : turbulent time scale
$\mathcal{S}^{-1}$: mean distortion time scale
$\mathcal{N}^{-1}$: mean stratification time scale

- $\mathcal{F}_s$ and $\mathcal{F}_n \geq 1$ → free decay → **Equilibrium regime**
  $\mathcal{F}_s$ or $\mathcal{F}_n \ll 1$ → **Rapidly Distorted Turbulence (RDT) regime**

- RDT results from :
  - stratification effects : stable or unstable (Rayleigh-Taylor)
  - compressions/expansions

Purpose : Derivation of a SGS model

Our purpose is to derive a subgrid-scale (SGS) model :

- coherent with RDT and equilibrium limits
- accounting for stratification and compression effects

- Main steps of the derivation :
  1. We discuss some properties of small scale turbulence in two idealized situations
     - Homogeneous isotropic turbulence submitted to a **rapid distortion**
     - Homogeneous isotropic turbulence submitted to a **slow distortion**
  2. We propose two SGS expressions compatible with each of these two asymptotic cases
  3. We treat the general case by combining these two expressions

$\Rightarrow$ **A mixed model is obtained**

Notations and general assumptions

- We consider a turbulent variable density flow governed by Navier-Stokes equations, with density ρ, velocity $\boldsymbol{u}$, pressure p, viscosity ν.
- The filtering operation is denoted by :

$$\langle Q \rangle_\Delta = \int G(\boldsymbol{r}, \Delta) Q(\boldsymbol{x} - \boldsymbol{r}) d\boldsymbol{r}$$

  $G(r, \Delta)$ is a commutative and isotropic filter.
- The subgrid stresses are defined by :
  $\tau_{ij} = \langle u_i u_j \rangle_\Delta - \langle u_i \rangle_\Delta \langle u_j \rangle_\Delta$, $\tau_{i\rho} = \langle \rho u_i \rangle_\Delta - \langle \rho \rangle_\Delta \langle u_i \rangle_\Delta$,
  $\tau_{\rho\rho} = \left\langle \rho^2 \right\rangle_\Delta - \langle \rho \rangle_\Delta^2$
  For later convenience, we collect the stresses in :

$$\Sigma_{ij}(\Delta) = \langle X_i X_j \rangle_\Delta - \langle X_i \rangle_\Delta \langle X_j \rangle_\Delta \text{ with } X_i = u_i \text{ for } i = 1, 2, 3 \text{ and } X_4 = \rho$$

- Statistical averages and fluctuations are denoted by :

$$\overline{Q} \text{ and } Q'$$

RDT limit

RDT limit and SGS models

- The impact of RDT on SGS modelling has been studied by several authors (i.e. Shao and Sarkar, 99; Chen *et al.* , 05)
- Most of these works consist in *a priori* evaluation of existing models against experiments or DNS (i.e. Liu *et al.* , 99) :
  - Smagorinsky-like models fail to reproduce the main behavior of small turbulent scales.
  - Scale-similarity and mixed models seem to perform better
- Fewer works explicitly use RDT theory to derive a SGS model :
  - *Laval & Dubrulle (01)* : stochastic Langevin model in spectral space based on RDT
  - *Li & Meneveau (04)* : exponential closure based on RDT with a restrictive "pressure released" assumptions
  - *Hill & Pantano* : vortex alignment based on strained RDT, but production/dissipation equilibrium is assumed

We aim to derive an algebraic model in physical space directly from RDT assumptions

WKB-RDT equations for variable density flows

- Main RDT assumptions :

$$\mathcal{F}_s(\kappa) = \frac{\omega_\kappa}{\mathcal{S}} \ll 1 \quad , \quad \mathcal{F}_n(\kappa) = \frac{\omega_\kappa}{\mathcal{N}} \ll 1$$

κ = wave number ; ω_κ = turbulent frequency at scale κ.

- Variable density Navier-Stokes equations can be linearized as :

$$\frac{\hat{D}\boldsymbol{M}}{Dt} = -\boldsymbol{A}\boldsymbol{M} - \boldsymbol{M}\boldsymbol{A}^\top \quad , \quad \text{with} \quad \frac{\hat{D}}{Dt}\cdot = \frac{\partial}{\partial t}\cdot - \kappa_k \frac{\partial \widetilde{U}_k}{\partial x_l}\frac{\partial}{\partial \kappa_l}\cdot$$

$\boldsymbol{M}$ is the 4×4 density-velocity spectral correlation tensor :

$$M_{ij}\delta(\boldsymbol{\kappa} + \boldsymbol{\kappa}') = \overline{\hat{X}_i'(\boldsymbol{\kappa})\hat{X}_j'(\boldsymbol{\kappa}')}$$

$\boldsymbol{A}$ is the interaction matrix :

$$A_{il} = \begin{cases} \frac{\partial \widetilde{U}_k}{\partial x_l}(\delta_{ik} - 2n_i n_k) - \frac{1}{2}\, div\, \widetilde{\boldsymbol{U}}\delta_{il} & , \quad i = 1,2,3 \text{ and } l = 1,2,3 \\ \frac{1}{\overline{\rho}}\frac{\partial \overline{P}}{\partial x_k}(\delta_{ik} - n_i n_k) & , \quad i = 1,2,3 \text{ and } l = 4 \\ \frac{1}{\widetilde{s}}\frac{\partial \widetilde{s}}{\partial x_k} & , \quad i = 4 \text{ and } l = 1,2,3 \\ -\frac{1}{2}\, div\, \widetilde{\boldsymbol{U}} & , \quad i = 4 \text{ and } l = 4 \end{cases}$$

$\widetilde{s}$ is the mean entropy and $\boldsymbol{n}$ is the unit wave vector : $\boldsymbol{n} = \boldsymbol{\kappa}/\kappa$.

Properties of the WKB-RDT solution for HIT

- **General case** :
  the WKB-RDT equations can be integrated as :
  $$\boldsymbol{M}(t) = \boldsymbol{H}(t,t')\boldsymbol{M}(t')\boldsymbol{H}^{\top}(t,t') \quad ,$$
  with $\boldsymbol{H}(t,t') = exp^{+}\left(-\int_{t'}^{t} \boldsymbol{A}(s)ds\right)$ along the path $\frac{d\kappa_i}{dt} = -\kappa_k \frac{\partial \widetilde{U}_k}{\partial x_i}$

- **Case of interest** :
  RDT applied to an initially homogeneous isotropic turbulence (HIT) with Kolmogorov spectrum. The solution becomes :
  $$\boldsymbol{M}(\boldsymbol{\kappa},t) = \frac{C_0 \widetilde{\varepsilon}^{2/3} \kappa^{-5/3}}{4\pi\kappa^2} \boldsymbol{b}(\boldsymbol{n},t) \qquad (1)$$
  where $\widetilde{\varepsilon}$ is the dissipation, C_0 is a constant, $\boldsymbol{b}$ is a matrix depending on the interaction history.

Consequences for the rapid distortion of an initial HIT

During the interaction, the inertial range scaling :
- is preserved
- determines the scaling of velocity anisotropy
- determines the scaling of density-velocity correlation

SGS model based on WKB-RDT

- The mean SGS stresses are linked to the turbulent spectrum $\boldsymbol{M}$ and the mean gradients :

$$\overline{\boldsymbol{\Sigma}}(\Delta) = \int \left[1 - \left|\hat{G}(\boldsymbol{\kappa}, \Delta)\right|^2\right] \boldsymbol{M}(\boldsymbol{\kappa}) d\boldsymbol{\kappa} + \Delta^2 \mathtt{G}^* \boldsymbol{C} \qquad (2)$$

$\hat{G}$: filter transfer function, $\mathtt{G}^*$: 2^{nd} order moment of the filter

$\boldsymbol{C}$: mean gradient tensor : $C_{ij} = \frac{1}{3} \frac{\partial \overline{X}_i}{\partial x_p} \frac{\partial \overline{X}_j}{\partial x_p}$

- Injecting the RDT solution (1) into the SGS stress tensor yields

$$\overline{\boldsymbol{\Sigma}}(\Delta) = \Delta^{\frac{2}{3}} \boldsymbol{B} + \Delta^2 \mathtt{G}^* \boldsymbol{C}$$

with : $\boldsymbol{B}(t) = c_0 \widetilde{\varepsilon}^{\frac{2}{3}} \int_0^\infty u^{-5/3} du \, \frac{1}{4\pi} \int_S \boldsymbol{b}(\boldsymbol{n}, t) dS$

- $\boldsymbol{B}$ and $\boldsymbol{C}$ are unknowns → 2 possibilities :
  - Make a short time expansion of B → functional model
  - Use super-grid information → structural model

  Literature tends to indicate that the second possibility is more appropriate

SGS model based on WKB-RDT

- Structural models are usually based on the Germano tensor.

$$\mathcal{L}_{ij} = \langle \langle X_i \rangle_\Delta \langle X_j \rangle_\Delta \rangle_{\tilde{\Delta}} - \langle \langle X_i \rangle_\Delta \rangle_{\tilde{\Delta}} \langle \langle X_j \rangle_\Delta \rangle_{\tilde{\Delta}}$$

- **Main issue** : $\mathcal{L}$ does not allow to separate mean/turbulent fields

⇒ **Proposed solution** : introducing a tensor, invariant under the addition of a constant mean gradient

$$\mathcal{G}_{ij} = \left(\frac{\partial \langle X_i \rangle_\Delta}{\partial x_p} - \frac{\partial \langle \langle X_i \rangle_\Delta \rangle_{\tilde{\Delta}}}{\partial x_p} \right) \left(\frac{\partial \langle X_j \rangle_\Delta}{\partial x_p} - \frac{\partial \langle \langle X_j \rangle_\Delta \rangle_{\tilde{\Delta}}}{\partial x_p} \right)$$

- $\mathcal{L}$ and $\mathcal{G}$ are linked to the spectrum and mean gradients by :

$$\overline{\mathcal{L}} = \left(\tilde{\tilde{\Delta}}^2 - \Delta^2 \right) \mathrm{G}^* \boldsymbol{C} + 2 \int \left[\left| \hat{G}^*(\kappa \Delta) \right|^2 - \left| \hat{G}^*(\kappa \tilde{\tilde{\Delta}}) \right|^2 \right] \boldsymbol{M} \, d\kappa$$

$$\overline{\mathcal{G}} = 2 \int \left| \hat{G}^*(\kappa \Delta) - \hat{G}^*(\kappa \tilde{\tilde{\Delta}}) \right|^2 \kappa^2 \boldsymbol{M} \, d\kappa$$

By injecting (1), we deduce the following model :

$$\overline{\boldsymbol{\Sigma}}(\Delta) = C_g(\mu) \Delta^2 \overline{\mathcal{G}} + C_l(\mu) \overline{\mathcal{L}}$$

For a sharp cut-off spectral filter : $\mu = \frac{\tilde{\tilde{\Delta}}}{\Delta}, C_g(\mu) = \frac{2}{\pi^2} \frac{\mu^2}{\mu^2 - 1}, C_l(\mu) = \frac{1}{\mu^2 - 1}$

Equilibrium limit

Equilibrium limit and SGS models

- Equilibrium limit plays a central role in many SGS models
- Usually :
  - Production/dissipation equilibrium in SGS equation
    → allows to set predefined model constants (Lilly, 1967)
- Other possibility :
  - Some theories predict equilibrium at the spectral level (Lumley, 1967; Ishihara *et al.* , 02)
  - These equilibrium spectra lead to Smagorinsky-like models (Li & Meneveau, 04)
- Applicability to shock tube context :
  - Most equilibrium theories are devoted to isovolume flows
  - Stratified variable density flows were dealt with by :
    - Yoshizawa (83), but erroneous equilibrium spectrum
    - Kaneda & Yoshida, but unknown constants
  - It seems that no derivation takes into account the effects of a mean compression/expansion

We aim to derive an equilibrium spectrum in presence of stratification and compression/expansion, with known constants

Canuto & Dubovikov spectral model

- Starting point = **Canuto & Dubovikov spectral model** (PoF, 1996)
  - Langevin model
  - Accounts for stratification and compression/expansion
  - Gives the evolution of the spectra of :
    - velocity E_{ij}, density Q, density/velocity F_i
  - 2 contributions :
    - Rapid contribution due to mean gradients $\approx$ RDT
    - Slow contribution $\approx$ transport/dissipation in spectral space → set by RNG techniques

- C&D model admits an **asymptotic solution** in the equilibrium regime

- Main hypotheses : High Froude Number + Stationarity

$$\frac{1}{\mathcal{F}_s(\kappa)} \sim \frac{1}{\mathcal{F}_n^2(\kappa)} \sim \epsilon \ll 1 \quad , \quad \frac{\partial}{\partial t} \cdot \sim \epsilon^2$$

Equilibrium asymptotic solution for Canuto's spectral model

- Asymptotic expansion : $\boldsymbol{x} = \boldsymbol{x}^{(0)} + \epsilon \boldsymbol{x}^{(1)} + \cdots$, ,

$$e_{ij}^{(0)} = C_0 \widetilde{\varepsilon}^{2/3} \kappa^{-5/3} \frac{\delta_{ij}}{3} \quad , \quad q^{(0)} = \sigma_t C_0 \widetilde{\varepsilon}_\rho \widetilde{\varepsilon}^{-1/3} \kappa^{-5/3}$$

$$\frac{e_{ij}^{(1)}}{e^{(0)}} = \left(\alpha_1 \frac{\mathcal{D}}{\omega_\kappa} + \alpha_2 \frac{\mathcal{N}^2}{\omega_\kappa^2} \right) \frac{\delta_{ij}}{3} - \alpha_3 \frac{\mathcal{S}_{ij}}{\omega_\kappa} - \alpha_4 \frac{\mathcal{N}_{ij}^2}{\omega_\kappa^2}$$

$$\frac{q^{(1)}}{q^{(0)}} = -\beta_1 \frac{\mathcal{D}}{\omega_\kappa} - \beta_2 \frac{1}{\omega_\kappa^2} \left(\frac{5}{8} \mathcal{N}^2 + \Omega^2 \right) \quad , \quad f_i^{(1)} = -\gamma_1 \frac{e^{(0)}}{\omega_\kappa} \frac{1}{\widetilde{s}} \frac{\partial \widetilde{s}}{\partial x_k}$$

e, f, q : modulus spectra ; $\alpha_*, \beta_*, \gamma_*, C_0, \sigma_t$: constants ; $\omega_\kappa = \widetilde{\varepsilon}^{1/3} \kappa^{2/3}$

$\Omega^2 = \overline{\rho}^2 \frac{\widetilde{\varepsilon}}{\widetilde{\varepsilon}_\rho} \frac{1}{\widetilde{s}} \frac{\partial \widetilde{s}}{\partial x_k} \frac{1}{\widetilde{s}} \frac{\partial \widetilde{s}}{\partial x_k}$; $\mathcal{D} = div\, \overline{\boldsymbol{u}}$

$\mathcal{N}_{ij}$: trace-free stratification tensor ; $\mathcal{S}_{ij}$: trace-free deformation tensor

Consequences

- Anisotropy is due to mean velocity gradients and stratification
- Stratification contribution scales as κ^{-3}
- Density flux scales as $\kappa^{-7/3}$
- Velocity divergence modifies density and energy spectrum

Equilibrium SGS model

- Equilibrium spectra are injected in equation (2) :

$$\frac{\overline{\tau_{ij}}}{2k_\Delta} = \frac{1}{3}\left[1 + a_\tau \frac{\mathcal{D}}{\omega_\Delta} + b_\tau \frac{\mathcal{N}^2}{\omega_\Delta^2}\right]\delta_{ij} - c_\tau \frac{\mathcal{S}_{ij}}{\omega_\Delta} - d_\tau \frac{\mathcal{N}_{ij}^2}{\omega_\Delta^2}$$

$$\frac{\overline{\tau_{\rho\rho}}}{\vartheta_\Delta} = 1 - a_\vartheta \frac{\mathcal{D}}{\omega_\Delta} - b_\vartheta \frac{\mathcal{N}^2}{\omega_\Delta^2} - c_\vartheta \frac{\Omega^2}{\omega_\Delta^2}$$

$$\overline{\tau_{i\rho}} = -\, a_\varphi \frac{k_\Delta}{\omega_\Delta}\eta_i$$

a_*, b_*, c_* and d_* are known constants

- k_Δ, ϑ_Δ ω_Δ can be determined with the tensor $\mathcal{G}$ → dynamic model :

$$k_\Delta \propto \Delta^2 \overline{\mathcal{G}}_{kk} \quad , \quad \vartheta_\Delta \propto \Delta^2 \overline{\mathcal{G}}_{\rho\rho} \quad , \quad \omega_\Delta \propto \sqrt{\overline{\mathcal{G}}_{kk}}$$

General case

Combining RDT and equilibrium limits

- Up to now, we derived :
  - a SGS model based on a spectrum M^{Eq} for $\mathcal{F} \gg 1$
  - a SGS model based on a spectrum M^{RDT} for $\mathcal{F} \ll 1$
- To treat the general case, we propose to arbitrarily decompose the mean stresses as :

$$\overline{\boldsymbol{\Sigma}}(\Delta) = \int_0^{\kappa_F} \left[1 - \left|\hat{G}(\boldsymbol{\kappa}, \Delta)\right|^2\right] \boldsymbol{M}^{RDT}(\boldsymbol{\kappa}) d\boldsymbol{\kappa} + \int_{\kappa_F}^{\infty} \left[1 - \left|\hat{G}(\boldsymbol{\kappa}, \Delta)\right|^2\right] \boldsymbol{M}^{EQ}(\boldsymbol{\kappa}) d\boldsymbol{\kappa} + \Delta^2 \mathrm{G}^* \boldsymbol{C}$$

- κ_F is a limit wave number such that :

$$\mathcal{F}(\kappa_F) = \mathcal{F}_0 \quad \Rightarrow \quad \kappa_F = \Delta^{-1} \left[\frac{\mathcal{F}_0}{\mathcal{F}_\Delta}\right]^{3/2}$$

$\mathcal{F}_0$ is a limit Froude number taken equal to 1
$\mathcal{F}_\Delta$ is the grid-scale Froude number :

$$\mathcal{F}_\Delta = \frac{\omega_\Delta}{\max(\mathcal{N}, \mathcal{S})} = \frac{\tilde{\varepsilon}^{1/3} \Delta^{-2/3}}{\max(\mathcal{N}, \mathcal{S})}$$

Mixed model

- The resulting model takes the form :

$$\overline{\tau}_{ij} = C_l(\mu)\overline{\mathcal{L}}_{ij} + C_g(\mu)\Delta^2\overline{\mathcal{G}}_{kk}\left\{\frac{\delta_{ij}}{3} + \xi_{\frac{5}{3}} b^{\mathcal{G}}_{ij} - \left[\left(1-\xi_{\frac{7}{3}}\right) A_\tau \frac{\mathcal{D}}{\omega_\Delta} + (1-\xi_3)\, B_\tau \frac{\mathcal{N}^2}{\omega_\Delta^2}\right]\frac{\delta_{ij}}{3} - \left(1-\xi_{\frac{7}{3}}\right) C_\tau \frac{\mathcal{S}_{ij}}{\omega_\Delta} - (1-\xi_3)\, D_\tau \frac{\mathcal{N}^2_{ij}}{\omega_\Delta^2}\right\}$$

$$\overline{\tau}_{\rho\rho} = C_l(\mu)\overline{\mathcal{L}}_{\rho\rho} + C_g(\mu)\Delta^2\overline{\mathcal{G}}_{\rho\rho}\left\{1 + \left(1-\xi_{\frac{7}{3}}\right) A_\vartheta \frac{\mathcal{D}}{\omega_\Delta} + (1-\xi_3)\left(B_\vartheta \frac{\mathcal{N}^2}{\omega_\Delta^2} + C_\vartheta \frac{\Omega^2}{\omega_\Delta^2}\right)\right\}$$

$$\overline{\tau}_{i\rho} = C_l(\mu)\overline{\mathcal{L}}_{i\rho} + C_g(\mu)\Delta^2\left\{\xi_{\frac{5}{3}}\overline{\mathcal{G}}_{i\rho} - \left(1-\xi_{\frac{7}{3}}\right)\frac{1}{1+\mu^{-\frac{2}{3}}}\frac{a_\varphi}{\omega_\Delta}\left[1 - A^S_\varphi \frac{\mathcal{D}}{\omega_\Delta} - B^S_\varphi \frac{\mathcal{N}^2}{\omega_\Delta^2}\right]\eta_i\right\}$$

A_*, B_*, C_*, D_* are constants

$b^{\mathcal{G}}_{ij} = \frac{\overline{\mathcal{G}}^\tau_{ij}}{\overline{\mathcal{G}}^\tau_{kk}} - \frac{\delta_{ij}}{3}$ is the anisotropy tensor of $\mathcal{G}^\tau$.

ξ_* are functions of $\mathcal{F}_\Delta$, with values between 0 and 1

Validation

Validation tests

- All subsequent tests are only preliminary
- They are necessary but certainly not sufficient to validate any model
- The tests consist in *a priori* comparisons against two DNS simulations
  - HIT/expansion wave interaction (RDT conditions)
  - Rayleigh Taylor Instability (Equilibrium conditions)
- Two aspects are examined :
  - Verification of the spectra on which the SGS model is based
  - Verification of the SGS model itself, but only for the velocity field !

DNS simulations with Triclade

(cf. M. Boulet and J. Griffond 10^{th} IWPCTM)
solves 3D compressible Navier-Stokes equations
+ concentration equations for mixtures of perfect gases.

- massively parallel implementation (MPI and MPI-I/O);
- object oriented conception (C++);
- several different high-order shock-capturing schemes.

Presently used scheme (hyperbolic part) :

- high (5^{th}) order one-step scheme (cf. V. Daru and C. Tenaud, JCP 193 (2004)) with uniform time and space accuracy;
- directional splitting;
- direct Euler solver (not Lagrange+projection);
- wave propagation method (cf. R.J. LeVêque);
- different Riemann solvers (cf. E.F. Toro);

Presently used scheme (elliptic part + sources) : operator splitting, 2^{nd} order treatment for viscous-diffusive terms and for sources.

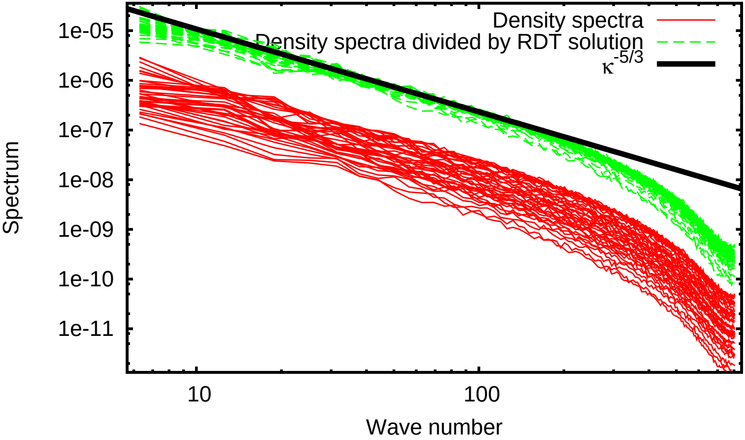
Interaction of HIT with a rarefaction wave

- Set-up :
  - A homogeneous isotropic turbulence with $\kappa^{-5/3}$ spectrum is impacted by a rarefaction wave
  - Initial values of turbulent field :
    $\overline{\rho'^2}/\overline{\rho}^2 \sim 3 \cdot 10^{-4}$, $\widetilde{k}/a^2 \sim 1.5 \cdot 10^{-3}$, $l_t \sim \frac{1}{3}$
  - RDT conditions are met :
    $\omega \sim 0.1, \mathcal{S} \sim 0.7 - 7 \quad \Rightarrow \quad Fr \sim 0.015 - 0.15$
  - Resolution : $256 \times 256 \times 896$
  - Domain size : $1 \times 1 \times 3.5$

RDT / DNS comparisons

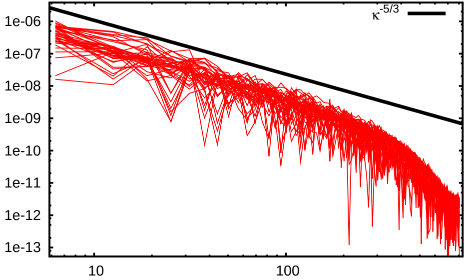
1D density spectra at $\omega t = 0.7$ at different locations in the expansion wave :

with and without non-dimensionnalization by the RDT solution

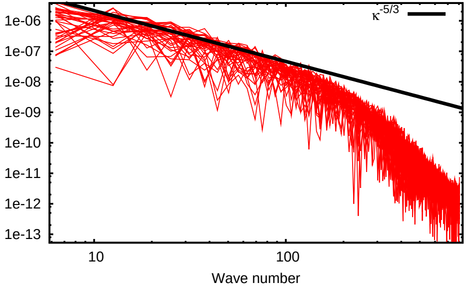


RDT : Mass flux and anisotropy inertial range scaling

$\kappa^{-5/3}$ scaling ($\neq$ equilibrium case) is verified

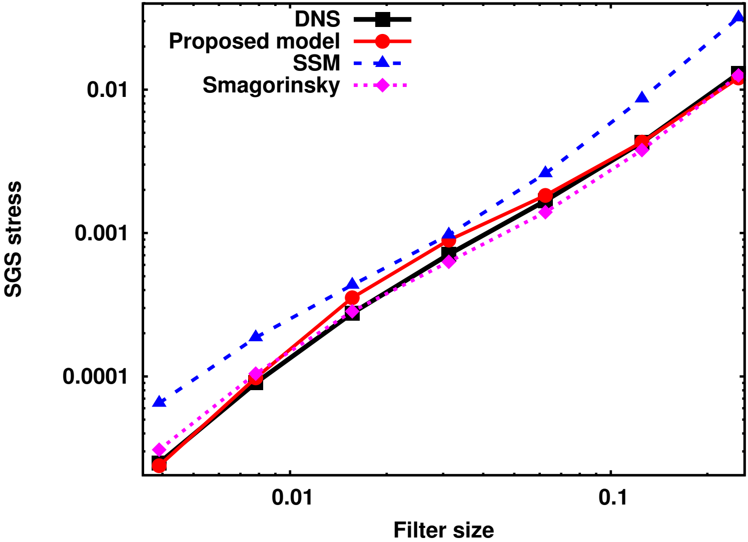


Anisotropy spectra ($E_{xx} - E_{zz}$) at $\omega t = 0.7$ at different locations in the expansion wave

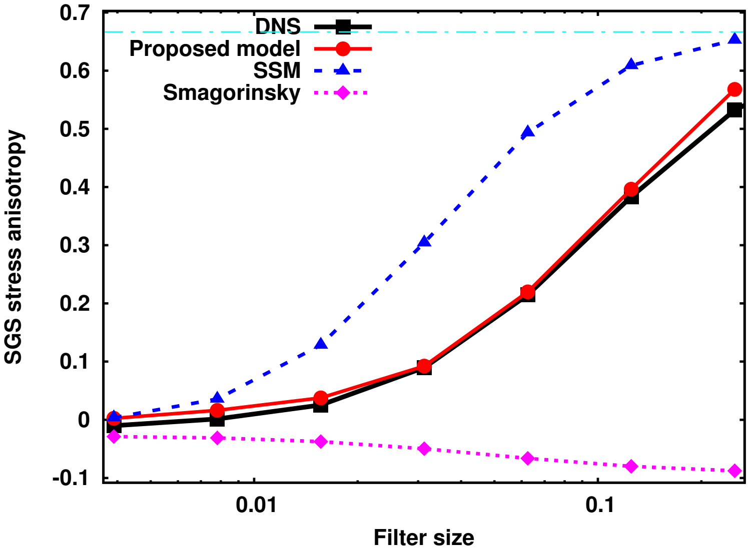


Density-velocity spectra at $\omega t = 0.7$ at different locations in the expansion wave :

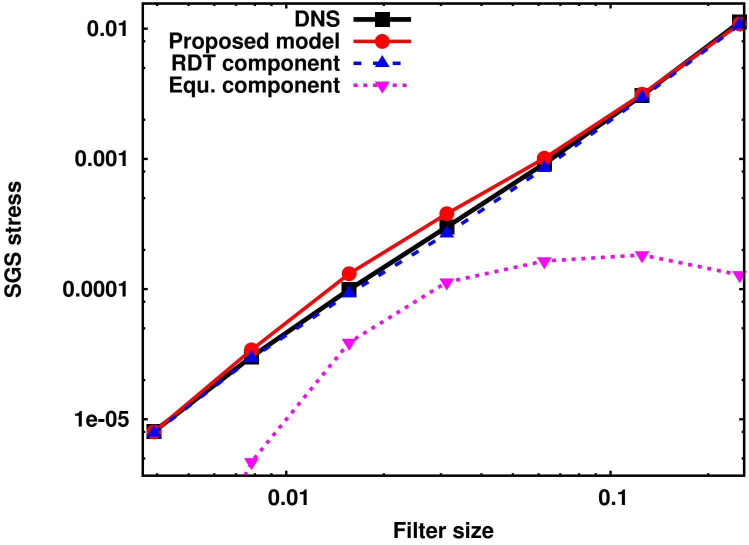
RDT : Mean subgrid scale energy $\overline{\tau}_{kk}$



RDT : Mean subgrid scale anisotropy $b_{11} = \frac{\overline{\tau}_{11}}{\overline{\tau}_{kk}} - \frac{1}{3}$

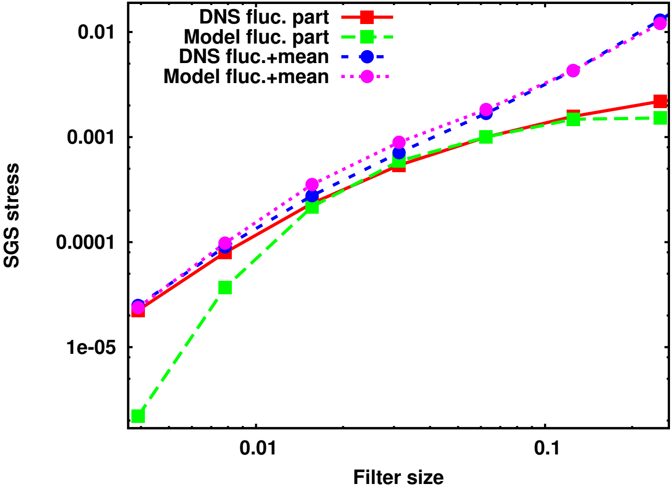


RDT : Structural and functional contributions to $\overline{\tau}_{kk}$

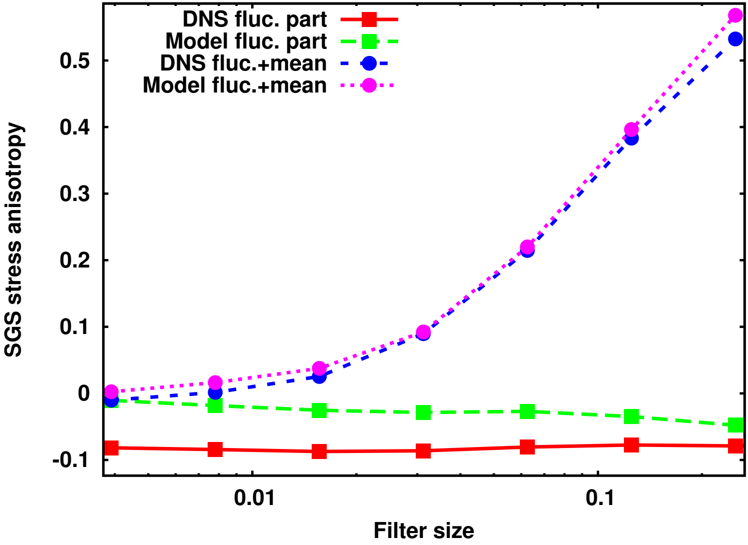


RDT : Fluctuating and mean subgrid scales for $\overline{\tau}_{kk}$

$$\overline{\mathbf{\Sigma}}(\Delta) = \int \left[1 - \left| \hat{G}(\boldsymbol{\kappa}, \Delta) \right|^2 \right] \boldsymbol{M}(\boldsymbol{\kappa}) d\boldsymbol{\kappa} + \Delta^2 G^* \boldsymbol{C} = \Sigma^{Fluc.} + \Sigma^{Mean}$$



RDT : Fluctuating and mean subgrid scales for b_{11}

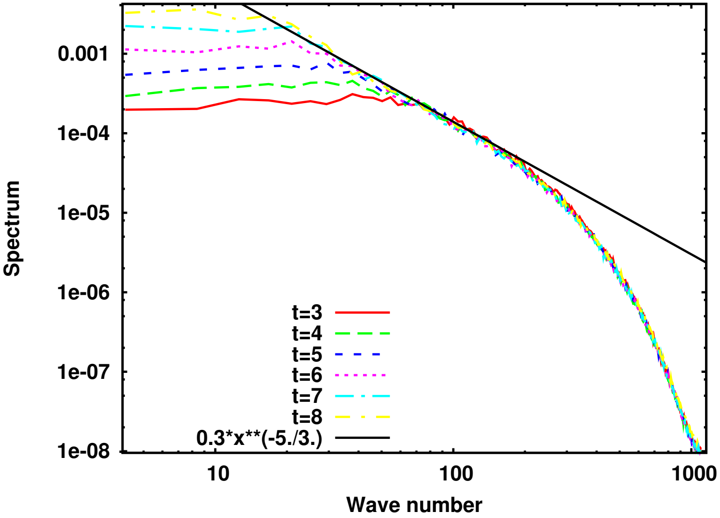


Rayleigh-Taylor turbulence

- Set up :
  - Atwood = 0.05
  - gravity = 1
  - Schmidt = 1
  - Periodic boundary conditions on the sides
  - Free walls at top and bottom
  - At initial time : perturbed velocity field at the interface
  - domain = $1 \times 1 \times 3$
  - Grid = $512 \times 512 \times 820$ with uniform grid spacing in $1 \times 1 \times 1$ domain
  - Calculation is stopped when the mixing zone reaches the non-uniform part of the grid
- Inertial range scaling appears.
  Equilibrium conditions are met for scales smaller than the integral scale

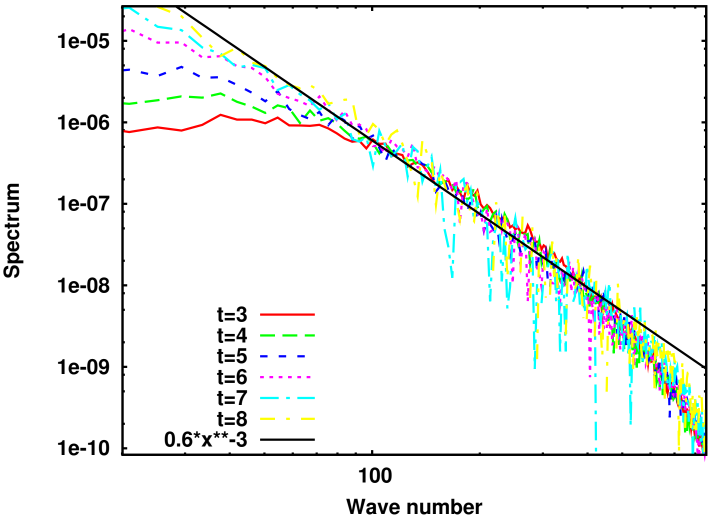
RTI : velocity spectrum

Vertical velocity 1D transverse spectrum divided by $\tilde{\varepsilon}^{\frac{2}{3}}$ at the mixing zone center



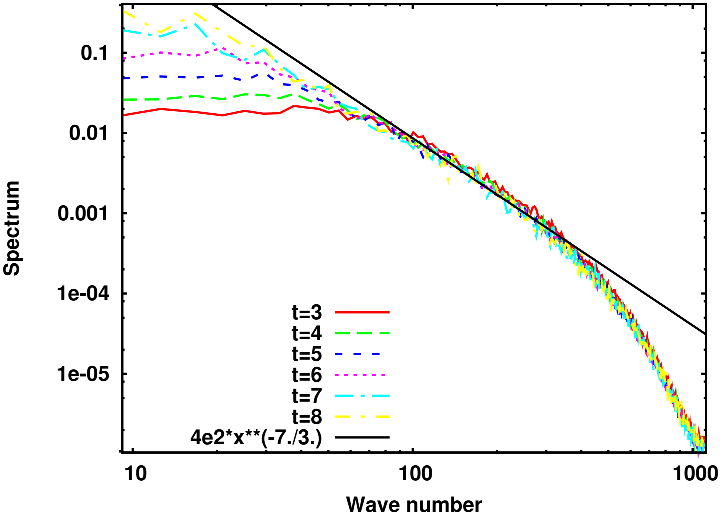
RTI : anisotropy spectrum

Velocity anisotropy 1D transverse spectrum $E_{zz} - E_{xx}$ divided by $\mathcal{N}^2$ at the mixing zone center



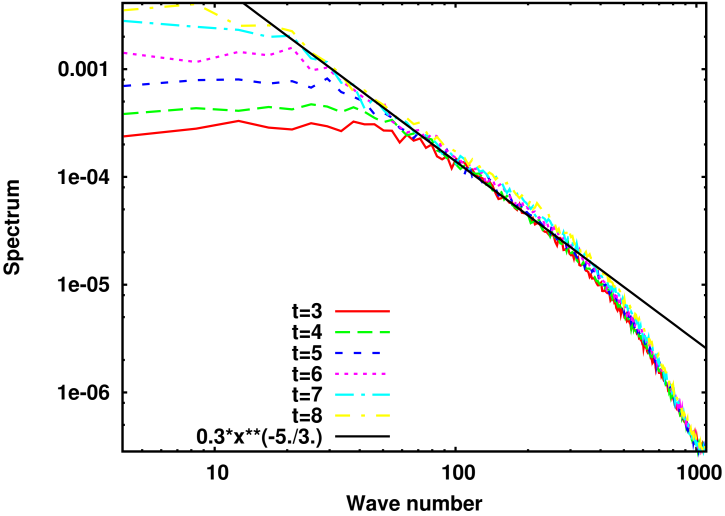
RTI : density flux spectrum

Density-vertical velocity 1D transverse spectrum $E_{\rho z}$ divided by $|\nabla \overline{\rho}|$ at the mixing zone center

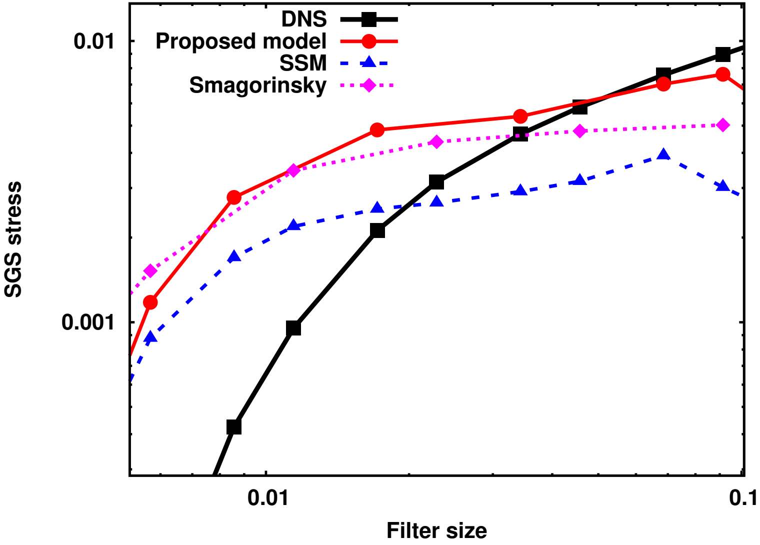


RTI : density spectrum

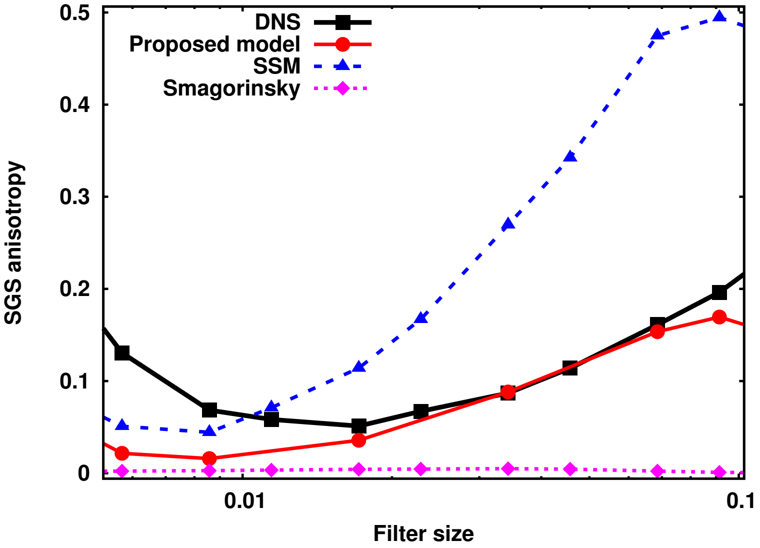
Density 1D spectrum divided by $\widetilde{\varepsilon}_\rho \widetilde{\varepsilon}^{-\frac{1}{3}}$ at the mixing zone center



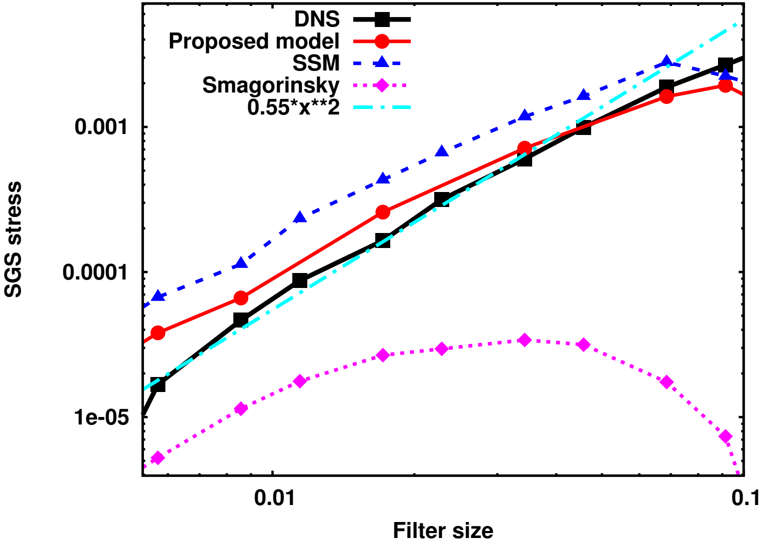
RTI : Mean subgrid scale energy $\overline{\tau}_{kk}$



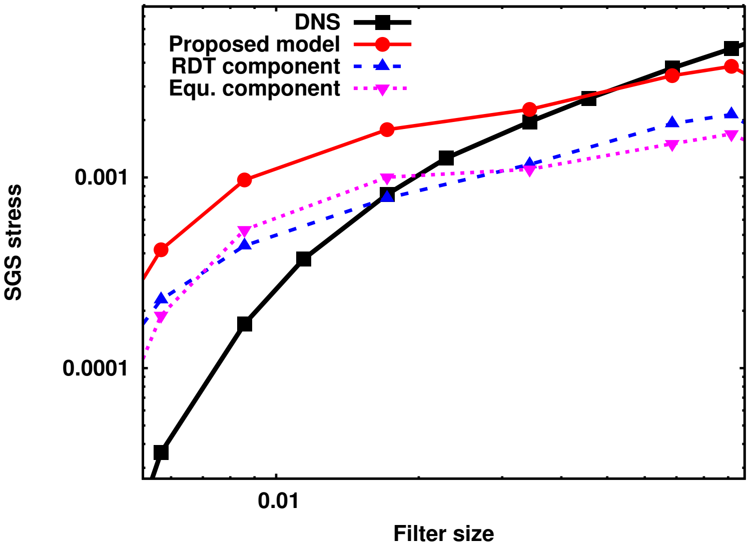
RTI : Mean subgrid scale anisotropy $b_{11} = \frac{\overline{\tau}_{11}}{\overline{\tau}_{kk}} - \frac{1}{3}$



RTI : Subgrid scale anisotropy $\overline{\tau}_{zz} - \overline{\tau}_{xx}$



RDT : Structural and functional contributions to $\overline{\tau}_{kk}$



Conclusions

- A subgrid scale model has been proposed
- This SGS was designed to match the two opposite limits of RDT and spectral equilibrium
- A few preliminary *a priori* validation tests have been performed
  - Expansion wave/HIT : RDT solution scalings were checked
  - Rayleigh-Taylor : equilibrium spectra were not contradicted by DNS results
  - The model seems to improve the prediction of anisotropy
- This is only the beginning of a validation process that will include more complete tests