Analytical theory for planar shock cylindrical and spherical focusing through perfect gas lens

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A gas lensing technique was documented in [P.E. Dimotakis and R. Samtaney, Phys. Fluids 18, 031705 (2006)] in order to convert a planar shock wave into a converging one. The lens is created by the interface between two different gases. The shape of this interface was determined by an iterative procedure and was found to closely match an ellipse or a hyperbola. A model was derived for cylindrical (two-dimensional) geometry and checked for shock waves going from a light gas to a heavier one.

In the present work, we reconsider the basic hydrodynamic equations for shock wave/gas interface interaction and revisit the gas lensing theory. For cylindrical and spherical (three-dimensional) geometries, we demonstrate that the shape of the lens is an ellipse for the light-to-heavy case and a hyperbola for the heavy-to-light case. We derive a simple formula to characterize theses shapes: the eccentricity e of the interface writes as $e = -W_t / W_i$, where W_t and W_i are the transmitted and incident shock wave velocities, respectively.

We confirm our theory with numerical simulations obtained with the Hesione code (see Figs. 1). Once a convergent shock wave is built, it can be used to study the Richtmyer-Meshkov instability (RMI) by adding a perturbed inner interface. We present computations of such instability. Firstly, the perturbed interface is subjected to the focusing shock wave and the RMI occurs. Secondly, after its focalization, the wave expands and re-shocks the interface which triggers a second RMI (See Figs. 2).

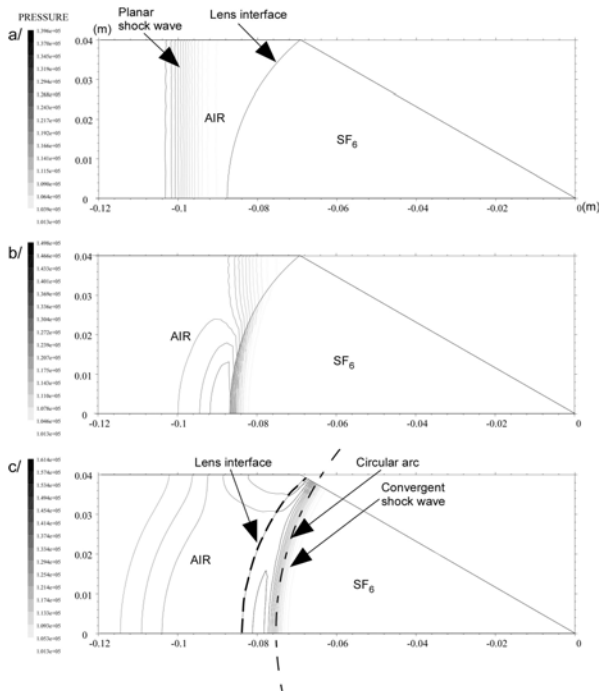

Figure 1: Numerical simulation of the focusing of a M0=1.15 planar shock wave through an Air/SF6 lens. Iso-level curves of pressure before (a), during (b) and after (c) the generation of the cylindrical transmitted shock wave.

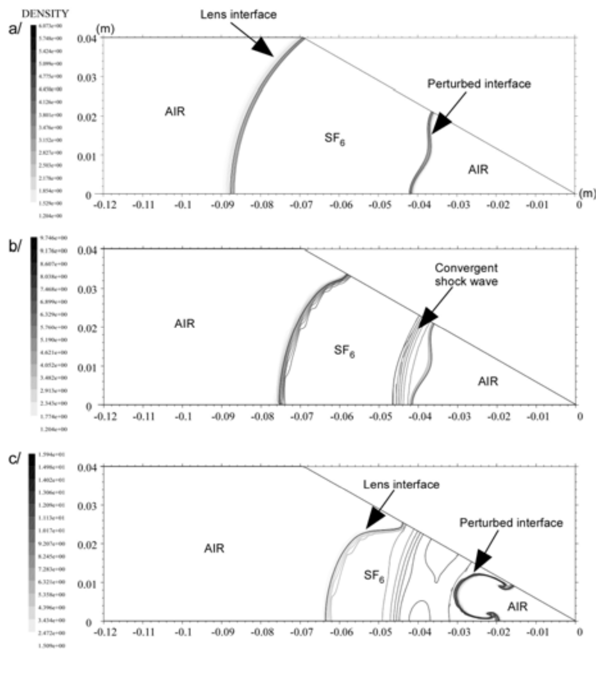

Figure 2: Numerical simulation of a Richtmyer-Meshkov instability due to the ingoing and outgoing cylindrical shock wave. Iso-level curves of density before the shock (a,b) and after the re-shock (c)of the perturbed inner interface.